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MOTION OF CONDUCTING BODIES IN A MAGNETIC FIELD

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This book represents a collection of articles on the theory of induction, magnetohydrodynamic (MHD) machines with a liquid metal working medium, the electromagnetic processes in an ideal, conducting MHD machine, and the higher spatial harmonics of the magnetic field of an induction MHD machine. It also discusses the transverse edge effect in plane, induction MHD machines, the longitudinal edge effect in linear MHD machines, the ponderomotive forces influencing conducting media in the traveling magnetic field of a cylindrical inductor, and a theory for the propagation of pulsed electromagnetic fields in moving, conductive media.

From the Editorial Board /5*

The book Dvizheniye provodyashchikh tel v magnitnom pole (Motion of Conductive Bodies in a Magnetic Field) which we offer to the reader is a survey of work on electromagnetic processes in magnetohydrodynamic (MHD) induction machines (generators, pumps) with working media of liquid metal.

The topics in this collection touch on the theory of MHD machines examined from the so-called electrodynamic approach, i.e., disregarding the magnetohydrodynamic effects in the machine channel.

The surveys include most of the papers published on the problem in question and several which were in the process of publication as the manuscript was being prepared for press; some original findings are also presented.

It is proposed in the immediate future to devote a separate work to a survey of magnetohydrodynamic phenomena in MHD induction machines.

Please send all requests and criticisms about the book to the Institute of Physics of the Academy of Sciences, Latvian SSR at 19 Turgenev St., Riga.

STATE OF THE THEORY OF MAGNETOHYDRODYNAMIC INDUCTION MACHINES WITH WORKING MEDIA OF LIQUID METAL

Ya. Ya. Liyelpeter

Magnetohydrodynamic (MHD) induction machines with a working media of liquid metal may be used as converters of the mechanical energy in the liquid metal flow into electrical energy (generators) or, on the contrary, as converters of electrical into mechanical energy (motors, pumps, brakes). Up till now liquid-metal MHD machines have found their chief use as pumps for transferring liquid metal. The use of these machines for generating electrical energy involves the development of methods for converting thermal energy into

* Numbers in the margin indicate pagination in the original foreign text.

energy of liquid metal flow. Elliot's proposal of a two-phase conversion cycle is noteworthy (Ref. 1); other systems are also well known. Comparatively few papers (Refs. 2, 3) have been published on liquid-metal MHD generators in contrast to plasma generators.

Approximate numerical evaluations of the basic characteristics of MHD induction generators indicate that at powers of the order of thousands of kilowatts their efficiency reaches 50-70%.

Papers by A. I. Vol'dek (Ref. 4) and N. M. Okhremenko (Ref. 5) give a synopsis of the theory of induction pumps. Below we shall scrutinize several aspects of the state of the theory, which are explained in less detail in the articles mentioned.

A large number of structural diagrams of MHD induction machines are known. The design of a specific MHD machine must always solve two problems: (1) selection of the optimum structural plan, and (2) finding the optimum relationships between the design variables for the given system. In optimizing the design of a specific machine, we must know the numerical relationships between the physical quantities determining its properties. In other words, we must analyze the characteristics of the specific machine with prescribed specific loads and geometrical dimensions.

From this it follows that the basic tasks in MHD induction machine theory are to analyze the properties of a set of structural diagrams at pre- /6
set specific loads and geometrical dimensions and to elaborate methods of optimizing dimensions and specific loads at the prescribed useful power. The first part of the task is by now in a comparatively well-developed state. The author gives a survey of the articles devoted to matters of optimization (Ref. 6). In the following we will touch only upon the first part of the theory of MHD induction machines.

The design of an MHD machine with liquid-metal working medium must unite the properties of electrical and hydraulic machines, and to the maximum degree must satisfy the specific demands made on both types of machine. The result of the latter is obviously that the channel of the optimum machine will have a cross-section varying over its length and a variable mean flow velocity even in the machine's active zone, while the electromagnetic and hydraulic processes in the channel will be complexly interconnected, making it impossible to study them separately. In this form the theory is at present completely undeveloped. In all the problems investigated, the channel cross-section is assumed to be constant along its length.

A concurrent solution of the electromagnetic-field and hydrodynamic equations in the last case is exceptionally difficult. Therefore, in practice, recourse is had to various sorts of approximations, e.g., the electromagnetic processes and the hydraulic phenomena are examined separately. Therefore, we may conventionally speak of the electromagnetic theory and the hydraulic theory of MHD machines, although they are necessarily interconnected. The first portion has, at present, been developed to the greatest extent. The second is for the time being in the initial stage of elaboration.

In electromagnetic theory, a liquid metal is regarded as a solid body, e.g., a strip, cylinder, or the like, which moves at a velocity equal to the average speed of the liquid. In this case, we must determine the electrodynamic force field, integral force, and energy (power) transmitted from the winding to the liquid metal (in the motor and brake regime), or in the other direction (in the generator regime). From these data we may compile the equivalent electrical circuit for the machine and its parameters, and may analyze its external characteristics and operating conditions.

Consideration of hydrodynamic effects in the simplest case is reduced to the fact that part of the electromagnetic pressure is expended on equalizing internal frictional forces in the liquid. Hence, the employed pressure is less than the electromagnetic pressure. Hydraulic losses in friction in this case are determined for the prescribed channel shape, due consideration being given to the effect of the magnetic field on hydraulic resistance.

In the approximation approach to MHD machine theory, one problem of /7
hydraulic theory is to define certain corrections to electromagnetic theory. In principle, this route may be justified to some degree, for we may assume that in the optimum machine the distribution of electromagnetic forces over the channel cross-section should not be very uneven. Nor will the distribution of averaged velocities substantially differ from the configuration during flow because of external pressure forces. The need for more detailed study of magnetohydrodynamic characteristics is, however, quite obvious.

Let us examine in somewhat more detail the state of MHD induction machine electromagnetic theory, about which the largest number of papers has been published. This part of the theory comprises the study of electromagnetic field structure in the working gap of the machine, as well as of processes in the magnetic and electrical circuits which determine the effective resistance and reactance of the windings. Magnetic and electrical field structure in the working gap depends on the structural shapes of the stators (inductors) and the channel configuration. Every structural layout of the machine has its own specific phenomena which have a substantial effect on the machine characteristics, e.g., in a plane induction pump an important role is played by a transverse edge effect which is lacking in a cylindrical pump.

Assuming the magnetic permeability of the magnetic circuit to be constant, we may describe the electromagnetic processes by a system of linear differential equations. When studying field structure in the working gap, we usually assume that one of the components of magnetic induction on the inductor surface is given, and we shall relate this component with the current in the machine windings.

The plane linear MHD machine has been studied in greatest detail. References 7-11 have discussed electromagnetic processes while neglecting the longitudinal and transverse edge effects, on the assumption that only the fundamental harmonics of the magnetic field is present. In this case, the fields depend only on a single coordinate directed across the channel. Cases are analyzed with conductive channel walls (Refs. 8, 9) and without them (Refs. 7, 10,

11). References 16-22 study the effect of the higher spatial harmonics of the magnetic field, which are caused by discrete spacing of the winding conductors and by serration of the inductor surface. This subject has also been touched upon in (Ref. 23). The theory of MHD induction machines may likewise partially use detailed studies of the higher harmonics in ordinary electrical machines. A comprehensive survey of this subject is found in Geller and Gamata's work (Ref. 40).

A comparatively large number of studies deal with the transverse edge effect in a plane MHD induction machine (Refs. 24-39). They solve problems which are stated in various ways and which examine the motion of a conduction band of finite width in a traveling magnetic field. The basic finding of these studies may be reduced to computing the coefficient of pressure attenuation and analyzing the influence of the transverse edge effect on characteristics of the machine. A new principle for simulating vortex fields in the conduction band is elaborated in (Ref. 33). References 34, 35, and 38 examine the transverse edge effect when there are shortcircuited buses in the channel of a plane MHD machine. /8

Despite the great number of published works, the question of transverse edge effect cannot be assumed to be thoroughly studied since many solutions (Refs. 26-28, 30, 31) are obtained for various initial conditions, while no general analysis or sufficiently comprehensive comparison of results with experimentation has been conducted.

A number of works have studied magnetic field structure of an inductor of finite length and the problems associated therewith of phase asymmetry in the winding (Refs. 41-46). The totality of these phenomena is usually called the longitudinal edge effect in the primary circuit. It has been ascertained that a break in the magnetic circuit leads to the generation of pulsed components of the magnetic field along with the traveling component. Methods have been proposed for balancing the pulsating fields. Distortion of field structure associated with motion of the conducting medium through the final zone of propagation of the traveling magnetic field (longitudinal edge effect in the secondary circuit) has been partially studied (Ref. 47). (Ref. 48) examines the edge effect in the primary and in the secondary circuit together.

A survey of longitudinal edge effect is found in (ref. 4).

The theory of cylindrical MHD induction machines is treated in fewer works than is that of plane machines, although this theory is more promising from the electromagnetic viewpoint (there is no transverse edge effect) since in practical computations the effect of channel curvature on field distribution may in the vast majority of cases be neglected, i.e., the problem may be regarded as a plane one. (Refs. 7, 49-54) scrutinize various problems for cylindrical MHD machines with constant channel cross-section without regard to longitudinal edge effect. (Ref. 49) computes the ponderomotive force acting on a conductive cylinder of finite length in an infinite cylindrical inductor and confirms the fact (which was observed experimentally previously) that force density peaks at a certain relative cylinder length.

The cited list of works on various problems of the motion of conducting bodies in a traveling magnetic field is not exhaustive. There are several experimental works studying magnetic field structure of an actual inductor in a MHD machine, the ponderomotive forces acting on solid conductive bodies, operating characteristics of the machine, and other effects (Refs. 55-59), but they are considerably fewer in number than the theoretical works. /9

When we speak of the MHD machine electromagnetic theory on the whole, we may note that there is a substantial number of works on this theory, but that by no means all matters have been investigated with adequate care. While the solutions themselves of electromagnetic field equations have been obtained for a large number of different cases, there is usually inadequate analysis of these solutions and comparison of them with other similar problems. The findings of the studies are rarely reduced to convenient computational coefficients and formulas defining the parameters of equivalent electrical circuits or conveying the basic energy characteristics of the machine - power, efficiency, power factor, etc. All this impedes practical utilization of research findings in the design of specific MHD machines.

One purpose of the present collection is to make a critical survey and compare the findings of papers on the electromagnetic theory of MHD induction machines in order to facilitate their use in design work. The collection adduces surveys of individual problems in the theory, e.g., on longitudinal and transverse edge effect, the influence of higher spatial field harmonics, etc. A separate range of questions is considered in the studies dealing with motion of conducting bodies in pulsed electromagnetic fields. A survey of these works is given in connection with their possible use in metering technology, e.g., in measuring the flow rate of a conducting liquid (Refs. 59-60). Various devices with traveling and pulsed magnetic fields are also applied for the same purpose (Refs. 61-63).

In conclusion let us briefly touch upon the state of the hydraulic theory of MHD induction machines.

Accurate solutions to problems of conductive liquid motion in a traveling magnetic field may be obtained only for the simplest cases of laminar flow, e.g., where velocity has only a single component, and magnetic field amplitude in the gap is either constant or varies in accord with a given law. Several such solutions have been published (Refs. 8, 64-69), but their results may be used merely to give a qualitative representation of the nature of the flow, since laminar flow conditions are very infrequently observed.

Of definite practical interest are the results of an experimental study of turbulent flow of a conducting fluid in a constant transverse magnetic field (Refs. 70-73). These experiments have been used to derive empirical relationships for the coefficient of hydraulic resistance in smooth and rough channels of constant cross-section, while taking into account the influence of the magnetic field. To design MHD induction machines, however, these findings must be utilized while taking into consideration the fact that the drag coefficient may be different in a traveling field. In this connection the results /10

of N. M. Okhremenko's article (Ref. 74) on the effect of a traveling magnetic field on the drag coefficient are very interesting.

Solutions are known for the distribution of electromagnetic fields in a conducting fluid moving with the velocity distribution given by a power law (Ref. 75).

Finally, there are certain experimental studies on fluid flow structure in the flow-through parts (channels) of MHD machines disregarding the magnetic field effect (Ref. 76). The chief purpose of these studies is to define the hydraulically optimum channel shapes - in particular their intake and outlet zones.

REFERENCES

1. Elliot, D. G. Two-fluid MHD Cycle for Nuclear Power Conversion. ARS Journal, 32, 1960.
2. Jackson, W. D., Pierson, E. S., Porter, R. P. Design Considerations for MHD Induction Generators. In: International Symposium on MHD Electrical Power Generation. Paris, Session 4c, Paper 61, July 6-11, 1964.
3. Brown, G. A., Jackson, W. D., Lee, K. S., Reid, M. H. MHD Power Generation with Liquid Metals. In: Proceedings of the Fifth Symposium of Engineering Aspects of Magnetohydrodynamics. M.I.T., Cambridge, Mass., April, 1964.
4. Vol'dek, A. I. Sostoyaniye i zadachi po rabotke induktsionnykh nasosov (State and Problems of Development of Induction Pumps). Trudy Tallinskogo Politekhnicheskogo Instituta, Seriya A (Proceedings of the Tallinn Polytechnical Institute, Series A), 197, 1962.
5. Okhremenko, N. M. Induktsionnyye nasosy s begushchim magnitnym polem (Induction Pumps with a Traveling Magnetic Field). Magnitnaya Gidrodinamika, 4, 1965.
6. Liyelpeter, Ya. Ya. Optimal'noye proyektirovaniye induktsionnykh magnitogidrodinamicheskikh mashin (obzor) (Optimum Planning of Magnetohydrodynamic Induction Machines [A Survey]). Magnitnaya Gidrodinamika, 1966 (in press).
7. Tyutin, I. A. Mekhanicheskiye sily v begushchem pole (Mechanical Forces in a Traveling Field). In: Voprosy Energetiki, 3 (Power Engineering Problems, 3). Izdatel'stvo AN Latv. SSR, 1955.
8. Tyutin, I. A. Elektromagnitnyye nasosy dlya zhidkikh metallov (Electromagnetic Pumps for Liquid Metals). Izdatel'stvo AN Latv. SSR, 1959.
9. Tyutin, I. A., Yankop, E. K. Elektromagnitnyye protsessy v induktsionnykh nasosakh dlya zhidkikh metallov (Electromagnetic Processes in Induction Pumps for Liquid Metals). Trudy Instituta Fiziki Akademii Nauk Latviyskoy SSR, 8, 1956.
10. Liyelpeter, Ya. Ya., Petrovich, R. A. K teorii ploskikh induktsionnykh nasosov (On the Theory of Plane Induction Pumps). Izvestiya Akademii Nauk Latviyskoy SSR, Seriya Fizicheskikh i Tekhnicheskikh Nauk, 1, 1964.
11. Watt, D. A. A Study in Design of Traveling Field Electromagnetic Pumps for Liquid Metals. Harwell, 1955.
12. Lopukhina, Ye. M. Issledovaniye asinkhronnogo dvigatelya s rotorom v

vide pologo tsilindra (Investigation of an Asynchronous Motor with Rotor in the Shape of a Hollow Cylinder). Elektrichestvo, 5, 1950.
13. Kochnev, E. K. K teorii ustroystv dlya elektromagnitnogo peremeshinvaniya rasplavlennogo metalla (On the Theory of Devices for Electromagnetic /11 Mixture of Melted Metal) Elektrichestvo, 7, 1959.
14. Veze, A. K., Liyelausis, O. A., Petrovicha, R. A., Ulmanis, L. Ya. Provodyashchiy sloy v begushchem elektromagnitnom pole odnostoronnego induktora (The Conducting Layer in the Traveling Electromagnetic Field of a Unidirectional Inductor). In: Voprosy Magnitnoy Gidrodinamiki. Izdatel'stvo AN Latv. SSR, 1963.
15. Veze, A. K., Krumin', Yu. K. Ob elektromagnitnoy sile, deystvuyushchey na beskonechnyy provodyashchiy sloy v begushchem magnitnom pole ploskikh induktorov (On the Electromagnetic Force Acting on an Infinite Conducting Layer in the Traveling Magnetic Field of Plane Inductors). Magnitnaya Gidrodinamika, 4, 1965.
16. Valdmanis, Ya. Ya., Mikel'son, Yu. Ya. Nakhozhdeniye elektromagnitnogo polya v beskonechnoy provodyashchey polose v pole ploskikh beskonechnykh induktorov (Locating the Electromagnetic Field in the Infinite Conducting Band in the Field of Infinite Plane Inductors). Izvestiya Akademii Nauk, Latviyskoy SSR, Seriya Fizicheskikh i Tekhnicheskikh Nauk, 1, 1965.
17. Mikel'son, Yu. Ya. Provodyashchiy sloy v begushchem elektromagnitnom pole dvukhstoronnego induktora (The Conducting Layer in the Traveling Electromagnetic Field of a Two-Directional Inductor). Ibid, 2, 1965.
18. Valdmanis, Ya. Ya., Kunin, P. Ye., Mikel'son, Yu. Ya., Taksar, I. M. Provodyashchiy sloy v begushchem elektromagnitnom pole dvukhstoronnego induktora (The Conducting Layer in the Traveling Electromagnetic Field of a Two-directional Inductor). Magnitnaya Gidrodinamika, 2, 1965.
19. Valdmanis, Ya. Ya. /Elektrodinamicheskiye sily, deystvuyushchiye na beskonechnuyu provodyashchuyu polosu v pole odnostoronnego induktora trekhfaznogo toka (Electrodynamic Forces Acting on the Infinite Conducting Band in the Field of a Unidirectional Three-Phase Current Inductor). Izvestiya Akademii Nauk Latviyskoy SSR, Seriya Fizicheskikh i Tekhnicheskikh Nauk, 1, 1965.
20. Mikel'son, Yu. Ya., Liyelpeter, Ya. Ya., Valdmanis, Ya. Ya. Vliyaniye vysshikh prostranstvennykh garmonik polya na elektrodinamicheskiye sily i dzhoulevy poteri v provodyashchey polose, dvizhushcheysya v begushchem magnitnom pole (Effect of High Spatial Field Harmonics on Electrodynamic Forces and Joule Losses in the Conducting Band Moving in a Traveling Magnetic Field). Ibid, 6, 1965.
21. Mikel'son, Yu. Ya., Sermons, G. Ya. Vliyaniye zubchatoy poverkhnosti induktora na raspredeleniye elektromagnitnogo polya v provodyaschey polose (Effect of a Serrated Inductor Surface on Electromagnetic Field Distribution in the Conducting Band). Ibid, 1, 1966.
22. Valdmanis, Ya. Ya., Kalnin', T. K. Elektromagnitnyy napor i poteri v induktsionnom nasose s dvizhushchimisya polyusami (Electromagnetic Pressure and Losses in an Induction Pump with Moving Poles). Ibid, 1966 (in press).
23. Ostroumov, G. A. Fiziko-matematicheskiye osnovy magnitnogo peremeshivaniya rasplavov (Physico-Mathematical Principles of the Magnetic Stirring of Melts). Metallurgizdat, 1960.
24. Vol'dek, A. I. Toki i usiliya v sloye zhidkogo metalla ploskikh induktsionnykh nasosov (Currents and Stresses in the Liquid Metal Layer in

Plane Induction Pumps). Izvestiya Vysshykh Uchebnykh Zavedeniy Electromekhanika, 1, 1959.

25. Ulmanis, L. Ya. K voprosu o krayevykh effektakh v lineynykh induktsionnykh nasosakh (On the Question of Edge Effects in Linear Induction Pumps). Trudy Instituta Fiziki Akademii Nauk Latviyskoy SSR, 8, 1956.
26. Okhremenko, N. M. Elektromagnitnyye yavleniya v ploskikh induktsionnykh nasosakh dlya zhidkikh metallov (Electromagnetic Phenomena in Plane Induction Pumps for Liquid Metals). Elektrichestvo, 3, 1960.
27. Okhremenko, N. M. Magnitnoye pole ploskogo induktsionnogo nasosa (The Magnetic Field of a Plane Induction Pump). Ibid, 8, 1964.
28. Okhremenko, N. M. Issledovaniye prostranstvennogo raspredeleniya magnitnykh yavleniy v induktsionnykh nasosakh (Investigation of Spatial Distribution of Magnetic Fields and Electromagnetic Phenomena in Induction Pumps). Magnitnaya Gidrodinamika, 1, 1965.
29. Yanes, Kh. I. Uchet vliyaniya vtorichnoy sistemy v lineynoy ploskoy magnitogidrodinamicheskoy mashine (Computation of the Effect of the Secondary System in a Linear, Plane Magnetohydrodynamic Machine). Trudy Tallinskogo Poli-Tekhnicheskogo Instituta, Seriya A, 197, 1962.
30. Veske, T. A. Resheniye uravneniy elektromagnitnogo polya ploskoy lineynoy induktsionnoy mashiny s uchetom vtorichnykh poperechnogo i tolshchinnogo krayevykh effektov (Solving the Equations of the Electromagnetic Field of a Linear Plane Induction Machine with Due Regard for Secondary Transverse and Thickness Edge Effects). Magnitnaya Gidrodinamika, 1, 1965.
31. Vilnitis, A. Ya. Raspredeleniye poley i tokov v provodyashchem tele pryamougol'nogo secheniya, pomeshchennom mezhdu dvumya beskonechnymi induktorami s sinusoidal'nym begushchim magnitnym polem (Distribution of Fields and Currents in a Conducting Body of Rectangular Cross-Section Situated Between Two Infinite Inductors with a Sinusoidal Traveling Magnetic Field). Izvestiya Akademii Nauk Latviyskoy SSR, Seriya Fizicheskikh i Tekhnicheskikh Nauk, 2, 1965.
32. Ulmanis, L. Ya. Fizicheskiye yavleniya pri induktsionnom vozdeystvii begushchego magnitnogo polya na sloy zhidkogo metalla (Physical Phenomena in the Inductive Action of a Travelling Magnetic Field on a Liquid Metal Layer). Author's abstract of candidate's dissertation. Riga, 1963. /12
33. Nitsetskiy, L. V. Printsip modelirovaniya elektricheskogo polya elektromagnitnykh nasosov v elektroliticheskoy vanne i na elektroprovodyashchey bumage (The Principle of Modeling the Electrical Field of Electromagnetic Pumps in an Electrolytic Bath and on Electrically Conductive Paper). In: Voprosy Magnitnoy Gidrodinamiki i Dinamiki Plazmy (Questions of Magnetic Hydrodynamics and Plasma Dynamics). Izdatel'stvo AN Latv. SSR, 1959.
34. Vol'dek, A. I., Yanes, Kh. I. Properechnyy krayevoy effekt v ploskikh induktsionnykh nasosakh pri kanale zhidkogo metalla s provodyashchimi stenkami (The Transverse Edge Effect in Plane Induction Pumps Having a Liquid Metal Channel with Conducting Walls). Ibid., 1962.
35. Vol'dek, A. I., Yanes, Kh. I. Poperechnyy krayevoy effekt v ploskom induktsionnom nasose s elektroprovodyaschim kanalom (The Transverse Edge Effect in a Plane Induction Pump with an Electrically Conductive Channel). Trudy Tallinskogo Politekhnicheskogo Instituta, Seriya A,

23, 197, 1962.
36. Kalnin', T. K., Petrovicha, R. A., Priyedniyeks, E. V. Napor i elektricheskiye poteri v sloye zhidkogo metalla yavnopolyusnykh induktsionnykh nasosov (Pressure and Electrical Losses in the Liquid Metal Layer of Phaneropolar Induction Pumps). Magnitnaya Gidrodinamika, 4, 1965.
37. Parts, I. R. Raspredeleniye elektromagnitnogo polya v zhidkometallicheskikh unipolyarnykh preobrazovatelyakh (Electromagnetic Field Distribution in Unipolar Liquid Metal Converters). Ibid., 3, 1965.
38. Parts, I. R. Raspredeleniye toka v zhidkom metalle ploskikh induktsionnykh nasosov pri nalichii korotkozamykayushchikh polos (Current Distribution in the Liquid Metal of Plane Induction Pumps in the Presence of Short-circuiting Bands). Ibid., 4, 1965.
39. Yanes, Kh. I., Veske, T. A. Uchet yavleniya vypuchivaniya magnitnogo polya iz nemagnitnogo zazora ploskogo lineynogo dvukhstoronnego induktora (Computing the Effect of Magnetic Field Bulge from the Nonmagnetic Gap of a Linear Plane Two-directional Inductor). Trudy Tallinskogo Tallinskogo Politekhnicheskogo Instituta, Seriya A, 214, 1964.
40. Geller, B., Gamata, V. Dopolnitel'nyye polya, momenty i poteri moshchnosti v asinkhronnykh mashinakh (Secondary Fields, Moments and Power Losses in Asynchronous Machines). Moscow-Leningrad, 1964.
41. Shturman, G. I. Induktsionnyye mashiny s razomknutym magnitoprovodom (Induction Machines with Open Magnetic Circuit). Elektrichestvo, 10, 1946.
42. Vol'dek, A. I. Pul'siruyushchiye sostavlyayushchiye magnitnogo polya induktsionnykh mashin i nasosov s razomknutym magnitoprovodom (Pulsed Components of the Magnetic Fields of Induction Machines and Pumps with Open Magnetic Circuits). Nauchnyye Doklady Vysshey Shkoly, Elektromekhanika i Avtomatika, 2, 1959.
43. Vol'dek, A. I. Iskazheniye simmetrii napryazheniy i tokov v induktsionnykh mashinakh i nososakh s razomknntym magnitoprovodom (Distortion of Voltage and Current Symmetry in Induction Machines and Pumps with Open Magnetic Circuits). Izvestiya Vysshykh Uchebnykh Zavedeniy, Electromekhanika, 5, 1960.
44. Vol'dek, A. I. Kompensatsiya pul'siruyushchego magnitnogo polya v askikhronnykh mashinakh i induktsionnykh nasosakh s razomknutym magnitoprovodom (Compensation of Pulsating Magnetic Field in Asynchronous Machines and Induction Pumps with Open Magnetic Curcuits). Electrichestvo, 4, 1965.
45. Vol'dek, A. I., Vyal'yamyae, G. Kh., Sillamaa, Kh. V., Tiysmus, Kh. A. Eksperimental'noye issledovaniye magnitnykh polyey v induktsionnykh mashinakh i nasosakh s razomknutym magnitroprovodom (Experimental Investigation of Magnetic Fields in Induction Machines and Pumps with Open Magnetic Circuits). Trudy Tallinskogo Politekhnicheskogo Instituta, Seriya A, 131, 1958.
46. Rashchepkin, A. P. Pole v zazore pri peremennoy lineynoy nagruzke obmotki induktsionnoy mashiny (The Field in the Gap with Alternating Linear Load of Induction Machine Winding). Magnitnaya Gidrodinamika, 3, 1965.
47. Vol'dek, A. I. Prodol'nyy krayevoy effekt vo vtorichnoy tsepi induktsionnykh nasosakh dlya zhidkikh metallov s razomknutym magnitoprovodom (The Longitudinal Edge Effect in the Secondary Circuit of Induction Pumps for Liquid Metals with Open Magnetic Circuits). Izvestiya

Vysshykh Uchebnykh Zavedeniy, Electromekhanika, 3, 1960.
48. Sudan, R. W. Interaction of a Conducting Fluid Stream with a Traveling Wave of Magnetic Field of Finite Extensions. J. Appl. Phys., 34, 3, 1963.
49. Kirshteyn, G. Kh. Provodyashchiy tsilindr konechnoy dliny, pomeshchennyy v begushcheye magnitnoye pole tsilindricheskogo induktora (A Conducting Cylinder of Finite Length Placed in the Traveling Magnetic Field of a Cylindrical Ferromagnetic Inductor). Izvestiya Akademii Nauk Latviyskoy SSR, Seriya Fizicheskikh i Tekhnicheskikh Nauk, 3, 1965.
50. Krumin', Yu.K. Zadacha o provodyashchem tsilindre, nakhodyashchemsya v begushchem magnitnom pole tsilindricheskogo induktora (The Problem of a Conducting Cylinder in the Traveling Magnetic Field of A Cylindrical Inductor). In: Elektromagnitnyye protsessy v metallakh (Electromagnetic Processes in Metals). SSR, 1959.
51. Krumin', Yu.K. Vychisleniye ponderomotornykh sil, deystvuyushchikh na provodyashchiy tsilindr v begushchem magnitnom pole tsilindricheskogo induktora (Computing the Ponderomotive Forces Acting on a Conducting Cylinder in the Traveling Magnetic Field of a Cylindrical Inductor). Izvestiya Akademii Nauk Latviyskoy SSR, Seriya Fizicheskikh i Tekhnicheskikh Nauk, 2, 1966.
52. Mikel'son, A.E., Nikolayev, V.N., Saulite, U.A. Opredeleniye radial'-/13
noy sostavlyayushchey begushchego magnitnogo polya v induktore tsilindricheskogo besserdechnikovogo nasosa (Determining the Radial Component of the Traveling Magnetic Field in the Inductor of a Coreless Cylindrical Pump). Ibid., 5, 1964.
53. Mikel'son, A.E., Saulite, U.A., Shkerstena, A.Ya. Issledovaniye tsilindricheskikh besserdechnikovykh nasosov (A Study of Coreless Cylindrical Pumps). Magnitnaya Gidrodinamika, 2, 1965.
54. Egle, I.Yu., Yankop, E.K. Analiticheskiy raschet elektromagnitnykh poley v induktsionnykh nasosakh tsilindricheskogo tipa (Analytic Calculation of Electromagnetic Fields in Induction Pumps of the Cylindrical Type). Uchenyye Zapiski Rizhskogo Politekhnicheskogo Instituta, 3, 7, 1962.
55. Vol'dek, A.I., Yanes, Kh.I. Eksperimental'noye issledovaniye ploskikh induktsionnykh nasosov (An Experimental Investigation of Plane Induction Pumps). In: Voprosy Magnitnoy Gidrodinami ki i Dinamiki Plazmy (Questions of Magnetic Hydrodynamics and Plasma Dynamics). Izdatel'stvo AN Latv. SSR, 1962.
56. Bleyer, E.B., Kalnin', T.K., Krishberg, R.R., Liyelpeter, Ya.Ya., Mikryukov, Ch.K., Okunev, G.A. Eksperimental'nyye issledovaniya ploskikh induktsionnykh nasosov (Experimental Studies of Plane Induction Pumps). Izvestiya Akademii Nauk Latviyskoy SSR, Seriya Fizicheskikh i Tekhnicheskikh Nauk 2, 1964.
57. Vol'dek, A.I., Vyal'yamae, G.Kh., Sillamaa, Kh.V., Tiysmus, Kh.A. Eksperimental'noye issledovaniye magnitnykh poley v induktsionniykh mashinakh i nasosakh dlya zhidkikh metallov s razomknutym magnitoprovodom (Experimental Investigation of Magnetic Fields in Induction Machines and Pumps for Liquid Metals with Open Magnetic Circuits). Trudy Tallinskogo Politekhnicheskogo Instituta, Seriya A, 131, 1958.
58. Yanes, Kh.I., Tiysmus, Kh.A., Veske, T.A., Liyn, Kh.A., Tammemyachi, Kh.A. Eksperimental'noye issledovaniye ploskikh induktsionnykh nasosov (Experimental Investigation of Plane Induction Pumps). Ibid., 197, 1962.

59. Sermons, G.Ya., Kalnin', R.K., Ginzburg, A.S. Avtorskoye svidetel'stvo (Author's Certificate) No. 168906 of February 16, 1963.
60. Sermons, G.Ya., Ginzburg, A.S. Avtorskoye svidetel'stvo (Author's Certificate) No. 166514 of July 27, 1963.
61. Kirshteyn, G.Kh., Rybakov, E.K. Avtorskoye svidetel'stvo (Author's Certificate) No. 169816 of October 4, 1963.
62. Kirshteyn, G.Kh., Rybakov, E.K., Ginzburg, A.S. Avtorskoye svidetel'stvo (Author's Certificate) No. 166516 of July 27, 1963.
63. Bespalov, G.P., Kalnin', R.K., Kirshteyn, G.Kh. Differentsial'nyy elektromagnitnyy izmerigel' skorosti elektroprovodyashchikh sred (A Differential Electromagnetic Device for Measuring Velocity of Electroconductive Media). Izvestiya Akademii Nauk Latv. SSR, Seriya Fizicheskikh i Tekhnicheskikh Nauk 1, 1965.
64. Tyutin, I.A. Vvedeniye v teoriyu induktsionnykh nasosov (Introduction to Induction Pump Theory). Trudy Instituta Fiziki Akademii Nauk Latv. SSR, 8, 1956.
65. Yankop, E.K. Raspredeleniye skorosti potoka i magnitogidrodinamicheskiye poteri dvleniya v gorlovine koaksial'nogo induktsionnogo nasosa (Flow Velocity Distribution and Magnetohydrodynamic Pressure Losses in the Entrance of a Coaxial Induction Pump). Uchenyye Zapiski Latviyskogo Gosudarstvennogo Universiteta, 10, 1957.
66. Liyelpeter, Ya.Ya. Gidrodinamicheskiye protsessy v kanale elektromagnitnogo induktsionnogo nasosa (Hydrodynamic Processes in the Channel of an Electromagnetic Induction Pump). In: Voprosy Magnitnoy Gidrodinamiki i Dinamiki Plazmy (Questions of Magnetic Hydrodynamics and Plasma Dynamics). Izdatel'stvo AN Latv. SSR, 1962.
67. Okhremenko, N.M. O nestatsionarnom techenii zhidkogo metalla v kanale ploskogo induktsionnogo nasosa (Non-stationary Flow of Liquid Metal in the Channel of a Plane Induction Pump). Ibid. 1962.
68. Okhremenko, N.M. Magnitogidrondinamicheskiye yavleniya v kanale ploskogo induktsionnogo nasosa s uchetom zatukhaniya elektromagnitnogo polya (Magnetohydrodynamic Effects in the Channel of a Plane Induction Pump Taking into Account Electromagnetic Field Damping). Ibid. 1962.
69. Mezhburd, V.I. Raspredeleniye skorosti zhidkogo metalla v kol'tsevom kanale pryamougol'nogo secheniya (Velocity Distribution of Liquid Metal in an Annular Channel of Rectangular Cross-Section). Trudy Tallinskogo Politekhnicheskogo Instituta, Seriya A, 214, 1964.
70. Murgatroyd, W. Experiments on Magneto-Hydrodynamics Channel Flow. Phil. Mag., 44, 13, 1953.
71. Harris, L. Magnitogidrodinamicheskiye techeniya v kanalakh (Magnetohydronamic Flows in Channels). Izdatel'stvo Inostrannoy Literatury 1963. /14
72. Branover, G.G., Liyelausis, O.A. Osobennosti vliyaniya poperechnogo magnitnogo polya na turbulentnyye techeniya zhidkogo metalla pri razlichnykh chislakh Reynol'dsa (Characteristics of the Effect of a Transverse Magnetic Field on Turbulent Flows of Liquid Metal at Various Reynolds Numbers). Zhurnal Technicheskoy Fiziki, 35, 2, 1965.
73. Branover, G.G. Ob odnoy prosteyshey raschetnoy zavisimosti magnitnoy gidravliki (A Very Simple Design Relationship in Magnetic Hydraulics). In: Voprosy Magnitnoy Gidrodinamiki (Problems in Magnetic Hydrodynamics). Izdatel'stvo AN Latv. SSR, 1964.

74. Okhremenko, N.M. Vliyaniye begushchego magnitnogo polya na gidravlicheskoye soprotivleniye turbulentnomu techeniyu provodyaschey zhidkosti v kanalakh (The Effect of a Traveling Magnetic Field on Hydraulic Resistance to Turbulent Flow of a Conducting Fluid in Channels). Ibid., 1963.
75. Dronnik, L.M., Tolmach, I.M. Techeniye elektroprovodyashchey zhidkosti v begushchem magnitnom pole pri stepennom zakone raspredeleniya skorosti po secheniyu kanala (Flow of an Electroconductive Fluid in a Traveling Magnetic Field with a Power Law Velocity Distribution over the Channel Cross-Section). Izvestiya Akademii Nauk SSR, Energetika i Transport, 3, 1964.
76. Branover, G., Liyelausis, O., Liyelpeter, Ya., Shcherbinin, E. Eksperimental'noye izucheniye gidravlicheskogo soprotivleniya protochnoy chasti elektromagnitnykh nasosov (An Experimental Study of the Hydraulic Resistance of the Flow-Through (Channel) Portion of Electromagnetic Pumps). Izvestiya Adademii Nauk Latviyskoy SSR, 4, 1963.

ELECTROMAGNETIC PROCESSES IN AN IDEAL, INDUCTION MHD MACHINE

A. K. Veze, L. Ya. Ulmanis

1. Introduction

The basic theory of induction MHD machines is concerned with the calculation of electromagnetic processes in an infinitely wide conductive band located in the traveling electromagnetic field of a flat inductor. If it is assumed that the conductive layer is infinitely long and that the sinusoidal traveling wave produced by the idealized inductor has only a basic harmonic component, such a problem may be readily solved analytically. The first problems of this type were solved in conjunction with the development of a theory for asynchronous engines (Ref. 1-2). Subsequently, such problems were solved for MHD machines. The results obtained were utilized for designing induction pumps (Ref. 3-6), but the theoretical formulas had to be refined by the introduction of certain empirical coefficients. These coefficients were employed to provide an approximate determination of the difference between the assumption advanced in the theoretical formulation of the problem and actual practice (Ref. 8, 18, 26).

The results obtained when such problems were solved were also applied when designing MHD generators, brakes, flow meters, electromagnetic mixers of metals in melting furnaces, chutes for transporting molten ferrous metals, and other equipment.

This article presents a summary of the solutions for electromagnetic processes in a conductive band located in a traveling magnetic field of a flat inductor, under the assumption that the dimensions of the device are infinite in the direction of motion of the field and in the direction in which the electric current passes.

2. Formulation of the Problem

A different number of layers with differing electroconductivity may be located in the operational zone of MHD machines. For example, an insulation layer, the channel wall, and in the middle a layer of electroconductive liquid are usually located on each side of the inductor in the clearance of flat induction pumps operating symmetrically, i.e., five layers in all. Pumps are also being designed for pumping two liquids at once; in such devices, the number of layers is greater. Therefore, it is advantageous to examine the problem in the general case, assuming the presence of many layers in the inductor. /16

A flat ideal inductor, producing a traveling magnetic field, may be represented in the form of one or two infinitely thin layers of current, whose

linear density (linear current load) has only one tangential component.

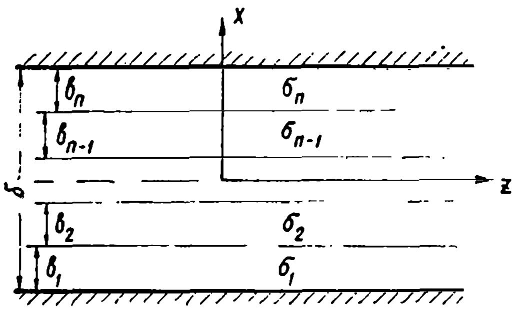

Figure 1

Arrangement of Conductive Layers in a Two-sided Inductor

When there are two layers of current, we shall call the inductor a two-sided inductor, and when there is one layer we shall it a one-sided inductor.

Our problem may be formulated as follows.

There are n parallel layers (Figure 1) in the traveling magnetic field of a flat inductor. The specific conductivity of these layers is σ_1, σ_2, ..., σ_n, and the thickness is b_1, b_2, ..., b_n, respectively. We shall assume that all the layers move at constant velocities with respect to the inductor equalling v_1, v_2, ..., v_n. In order to compute the velocity of each layer, let us introduce the following notation:

$$s_1 = \frac{v_э - v_1}{v_э};$$

$$s_2 = \frac{v_э - v_2}{v_э};$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$s_n = \frac{v_э - v_n}{v_э},$$

where $v_э = 2\tau f$ is the velocity of the traveling magnetic field with respect to the inductor (τ - polar division of the inductor, f- frequency of the current supplying the inductor). We may assume that the magnetic permeability of /17
all the layers, except the magnetic circuit of the inductor, is constant, and equals $\mu_0 = 4\pi \cdot 10^{-7} \frac{h}{m}$. It equals μ = Const. in the magnetic circuit of the inductor. The pole division of the inductor, and also velocity of the field along both of its sides are assumed to be the same.

The magnetic field **H** the vector potential **A** and the electric field E have the following components in the given case:

$$\mathbf{H}\{H_x;\ 0;\ H_z\};$$

$$\mathbf{A}\{0;\ A_y;\ 0\};$$

$$\mathbf{E}\{0;\ E_y;\ 0\}.$$

From this point on, we shall designate A_y simply by A and E_y by E.

The following differential equation may be solved for each of the k layers

$$\frac{\partial^2 A_k}{\partial x^2} + \frac{\partial^2 A_k}{\partial z^2} = \mu\sigma_k \frac{\partial A_k}{\partial t}. \qquad (1)$$

In the case under consideration, all the electromagnetic quantities have a sinusoidal dependence on ωst - az, and they may be expressed by means of functions having the following form

$$\dot{\chi} = \dot{\chi}_0(x)\, e^{i(\omega s t - \alpha z)},$$

where $\dot{\chi}$ is the corresponding electromagnetic quantity. Equation (1) then assumes the following form

$$\frac{\partial^2 \dot{\chi}_{0k}}{\partial x^2} - \beta_k^2 \dot{\chi}_{0k} = 0, \qquad (2)$$

where

$$\beta_k = \sqrt{\alpha^2 + i\mu\sigma_k\omega s_k} = \psi_{1k} + i\psi_{2k}; \qquad (3)$$

$$\psi_{1k} = \sqrt{\frac{\sqrt{\alpha^4 + \mu_k\sigma_k\omega s_k} + \alpha^2}{2}} = \alpha\sqrt{\frac{\sqrt{\varepsilon_k^2 + 1} + 1}{2}} = \alpha\bar{\psi}_{1k}; \qquad (4)$$

$$\psi_{2k} = \sqrt{\frac{\sqrt{\alpha^4 + \mu_k\sigma_k\omega s_k} - \alpha^2}{2}} = \alpha\sqrt{\frac{\sqrt{\varepsilon_k^2 + 1} - 1}{2}} = \alpha\bar{\psi}_{2k}; \qquad (5)$$

$$\varepsilon_k = \frac{\mu_k\sigma_k\omega s_k}{\alpha^2}. \qquad (6)$$

On surfaces dividing adjacent layers, the magnetic and electric fields must satisfy the following boundary conditions:

/18

$$\begin{aligned}
&B_{x1} = B_{x2};\quad H_{z1} = H_{z2}\ \text{for}\ x = b_1 - \frac{\delta}{2};\\
&B_{x2} = B_{x3};\quad H_{z2} = H_{z3}\ \text{for}\ x = b_1 + b_2 - \frac{\delta}{2};\\
&\cdots\\
&B_{xk} = B_{x(k+1)};\quad H_{zk} = H_{z(k+1)}\ \text{for}\ x = b_1 + \ldots + b_k - \frac{\delta}{2}
\end{aligned} \qquad (7)$$

(where δ is the distance between both sides of the inductor).

We shall define one of the components of the traveling magnetic field on the inductor surface - i.e., in the case of $x = \pm\frac{\delta}{2}$. Practical computations have shown that it is more advantageous to define the tangential component of the field, since - if the magnetic permeability of the inductor magnetic circuit is μ = ∞ - numerically it equals the linear current load of the inductor, and does not depend on processes occurring in the electroconductive layers.

Thus, in the general case the boundary conditions on the inductor surface

may be formulated as follows:

$$H_z = H_{01} e^{i(\omega s t - \alpha z)} \text{ for } x = \frac{\delta}{2};$$
$$H_z = -H_{02} e^{i(\omega s t - \alpha z + \varphi)} \text{ for } x = -\frac{\delta}{2}. \quad (8)$$

Here ϕ is the angle at which the phase shifts between the magnetic fields on both sides of the inductor, and it assumes any value in the general case. In practice, $\phi = 0$ customarily holds - i.e., the windings on both sides of the inductor are switched on concurrently. In the literature, with the exception of several articles (Ref. 9, 10), it is this case which is primarily investigated.

In all, there are 2n boundary conditions which are requisite in order to determine 2n integration constants. If it is assumed that in the case of $x = \frac{\delta}{2}$ the linear current density equals zero, we have a one-sided inductor.

If the problem is symmetrical with respect to the x = 0 plane - i.e., $b_k = b_{n-(k-1)}$; $\sigma_k = \sigma_{n-(k-1)}$ and $\phi = 0$ - the number of layers n is odd, since for even n there would have to be two adjacent layers with different physical parameters in the middle of the clearance. However, in this case the problem is non-symmetrical. The solution of the symmetrical problem in the mean band is expressed by the even or odd function of x depending on whether the /19 desired quantity has even or odd symmetry. However, solutions in symmetrical bands, which do not come in contact with each other, are characterized by a sign before the coordinate x for quantities with even symmetry, and are characterized by a sign before the coordinate x and before every expression for quantities with odd symmetry.

There is no necessity of finding a solution for every layer when solving the symmetrical problem. We may find a solution for only one half of the symmetrical region, assuming that there is a tangential component which equals zero in the middle of the clearance. The solution for the other half of the symmetrical region may be found by using the relationships presented above between the solutions in symmetrical layers. It is simpler to solve the problem by employing symmetry, since we must determine only n + 1 constant, instead of 2n constants.

The force lines of the magnetic field may be determined by solving the differential equation

$$\frac{dx}{dz} = \frac{\mathrm{Re}\dot{H}_x}{\mathrm{Re}\dot{H}_z}. \quad (9)$$

The force acting upon the conductive band is composed of a constant and pulsating components. The latter pulsates with the double frequency of the traveling magnetic field. The force may be computed analytically by multiplying the instantaneous values of current density and magnetic induction. Pulsations of the force density may have an influence upon the hydrodynamic processes in liquids located in a traveling magnetic field. When the ponderomotive forces

in induction pumps and other MHD machines are investigated, the pulsation of the force component is usually disregarded, and its constant component is computed, which equals the force averaged over time.

In the problem under consideration, there are two components of the force density - the tangential component f_z and the normal component f_x. Their constant components may be computed according to the following formulas

$$f_x = \frac{1}{2}\,\mathrm{Re}(j_y B_z^*) \tag{10}$$

and

$$f_z = -\frac{1}{2}\,\mathrm{Re}(j_y B_x^*). \tag{11}$$

They depend only on the x-coordinate. The electromagnetic pressure per unit of length p_z in the k-th layer, which is produced by the traveling magnetic field - i.e., the density of the tangential force component, is /20

$$p_{zk} = -\frac{1}{2b_k}\int_{x_0}^{x_0+b_k} \mathrm{Re}(j_y B_x^*)\,dx, \tag{12}$$

and the mean density of the normal force component is

$$p_{xk} = \frac{1}{2b_k}\int_{x_0}^{x_0+b_k} \mathrm{Re}(j_y B_z^*)\,dx. \tag{13}$$

In the symmetrical case, the tangential component of the force density has even symmetry, and the normal component has odd symmetry. This means that the normal component of the resulting force equals zero in the symmetrical case. It only contracts the body, but does not displace it.

The power transmitted to the conductive band from the inductor by means of the traveling magnetic field may be expressed by means of the normal component of the Poynting vector, which expresses the density of the electromagnetic energy flux per unit of time:

$$\dot{S}_x = \frac{1}{2}[E_y H_z^*]. \tag{14}$$

The active power transmitted to the body through a unit of surface area equals the real part of the the complex $\dot{S}_x$, and the reactive power equals the imaginary part of the complex $\dot{S}_x$.

If the coordinate system is related to the moving conductive band, then the Poynting vector may express that portion of the energy which is converted into Joule heat losses. If the coordinate system is related to the inductor, $\dot{S}_x$ expresses that portion of the electromagnetic energy which is converted into mechanical energy.

The studies (Ref. 1-25) solved different problems of the type examined above. Let us analyze certain cases, which are of the greatest interest.

3. Electromagnetic Phenomena in a Traveling Field of a One-Sided Inductor

(a) Conductive Half-Space

In a study by I. M. Kirko (Ref. 11) the simplest problem of this type was investigated - the motion of conductive half-space in a traveling magnetic filed.

A conductive medium with the specific electric conductivity σ fills the half-space $x > 0$, and moves at a constant velocity v along the direction of motion of a traveling magnetic field (Figure 2), which is produced by an infinitely thin current layer located on the surface $x = 0$. In the case of $x < 0$, it is assumed that the half-space is filled by an ideal ferromagnet with $\mu = \infty$ and $\sigma = 0$. /21

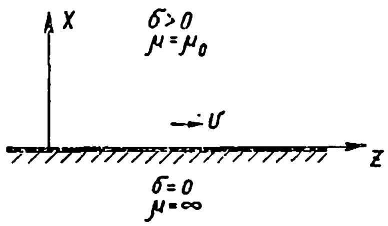

Figure 2

Half-Space Above a One-Sided Inductor

The solution of Maxwell equations provides the following values for the components of the electromagnetic field in conductive half-space:

$$\dot{B}_{xm}=\frac{i\alpha\mu_0 A_0}{\beta}e^{-\beta x}, \tag{15}$$

$$\dot{B}_{zm}=\mu_0 A_0 e^{-\beta x}, \tag{16}$$

$$\dot{E}_m=\frac{i\omega\mu_0 s A_0}{\beta}e^{-\beta x}, \tag{17}$$

$$j_m=\frac{i\omega\mu_0\sigma s A_0}{\beta}e^{-\beta x}. \tag{18}$$

Thus, all of the field components and the current density decrease according to an exponential law as one recedes from the surface $x = 0$.

At the distance $\frac{1}{\Psi_1}$ from the surface $x = 0$, the magnetic field decreases by a factor of e - i.e., this quantity represents the depth to which the traveling magnetic field penetrates (Ref. 3)

$$h_0=\frac{1}{\psi_1}=\frac{1}{\alpha}\sqrt{\frac{2}{\sqrt{1+\varepsilon^2}+1}}. \tag{19}$$

In the special case, when $\varepsilon \to 0$ - i.e., when the conductive medium is absent or fs = 0 - we have

$$h_0=h_r=\frac{1}{\alpha}.$$

In this case, the pole division of the inductor τ - the quantity which is contained in the dimensionless frequency ε - determines the depth to which the traveling magnetic field penetrates. This phenomenon is arbitrarily designated as the geometric surface effect.

The surface effect in a traveling magnetic field is characterized /22 by the fact that the planes of the same phase of the field are not perpendicular to the x > 0 plane, but are inclined toward it at the following angle

$$\varphi = \tan^{-1}\frac{1}{\overline{\psi}_2}.$$

We obtain the amplitude of the normal component of magnetic induction from expression (15)

$$B_{xm} = \frac{\mu_0 A_0}{\sqrt[4]{1+\varepsilon^2}} e^{-\psi_1 x} \tag{20}$$

and the tangent of the angle at which the phase shifts x < 0, as compared with the phase surface density of the current A_0:

$$\tan\varphi_x = \frac{\overline{\psi}_1 \cos\psi_2 x - \overline{\psi}_2 \sin\psi_2 x}{\psi_1 \sin\psi_2 x + \overline{\psi}_2 \cos\psi_2 x}. \tag{21}$$

The ratio of the amplitudes of the tangential and normal induction components increases with an increase in ε;

$$\frac{B_z}{B_x} = \sqrt[4]{1+\varepsilon^2}. \tag{22}$$

The angle at which the phase of the tangential induction component shifts changes proportionally to the x coordinate;

$$\varphi_z = -\psi_2 x. \tag{23}$$

All of the quantities investigated are determined by the parameter ε.

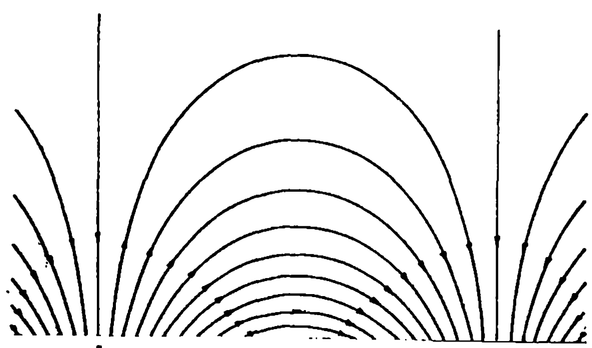

Figure 3

Force Lines of the Traveling Magnetic Field Above a One-Sided Inductor in the Case of ε = 0

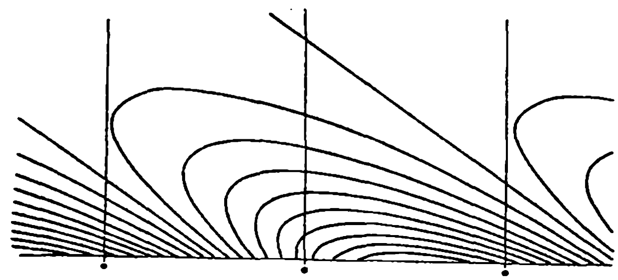

Figure 4

Force Lines of a Traveling Magnetic Field Above a One-Sided Inductor in the Case of $\pi^2\varepsilon = 50$

Figures 3 and 4 illustrate the magnetic force lines in an infinite half-space above a one-sided inductor for two values of ε, which were calculated by Yu K. Krumin. Similar problems were investigated by Schilder in (Ref. 23).

The parameter ε also determines the density of the pondermotive force averaged over time and the electromagnetic power;

$$f_z=\frac{1}{2}\,\alpha\mu_0 A_0^2\frac{\varepsilon}{\sqrt{1+\varepsilon^2}}\,e^{-2\psi_1 x}, \tag{24}$$

$$f_x=\frac{1}{2}\,\psi_2\mu_0 A_0^2\,\frac{\varepsilon}{\sqrt{1+\varepsilon^2}}\,e^{-2\psi_1 x}, \tag{25}$$

or /23

$$\frac{f_x}{f_z}=\overline{\psi}_2; \tag{26}$$

$$P_a=\frac{1}{2}\cdot\frac{\omega}{\alpha}\mu_0 A_0^2\frac{\overline{\psi}_2}{\sqrt{1+\varepsilon^2}};$$

$$P_r=\frac{1}{2}\cdot\frac{\omega}{\alpha}\,\mu_0 A_0^2\,\frac{\overline{\psi}_1}{\sqrt{1+\varepsilon^2}}. \tag{27}$$

The power factor of an idealized energy convertor, without allowance for the leakage flux of the winding, is

$$\cos\varphi=\frac{\overline{\psi}_2}{\overline{\psi}_1}. \tag{28}$$

The dimensionless quantity ε is an important characteristic of the /24 electromagnetic processes in problems of the type which we are considering. It was called electromagnetic slipping in a study by Kh. I. Yanes (Ref. 19). This quantity may be also regarded as the magnetic Reynolds number

$$\mathrm{Re}_m = \frac{ud}{\nu_m},$$

in which the velocity u is represented by the relative velocity of motion of the traveling magnetic field and of the conductive medium $\frac{\omega s}{\alpha}$. The distance $\frac{1}{\alpha} = \frac{\tau}{\pi}$, is assumed to be a characteristic linear dimension, and the magnetic viscosity equals $\nu_m = \frac{1}{\mu_0 \sigma}$.

The quantity ε changes sign when the slipping sign changes: in the braking and pumping regimes $\varepsilon > 0$, and in the generator regime of the MHD machine $\varepsilon < 0$.

(b) Conductive Layer in the Field of a One-Sided Inductor

The articles (Ref. 12, 13) investigated the following problem concerning three layers: the conductive layer which moves at a constant velocity $2\tau f (1 - s)$ with respect to the inductor is located at the distance δ from the one-sided inductor of a traveling magnetic field (thickness of the conductive layer, b; specific conductivity, σ; specific conductivity of the regions I and III equals zero). The non-conductive layer behind the conductor is not limited (Figure 5).

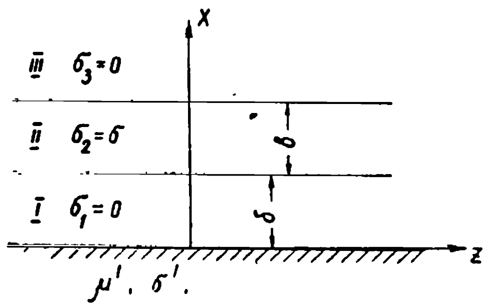

Figure 5

Solution of the Problem Regarding Three Layers in the Field of a One-Sided Inductor

Let us analyze certain relationships obtained when the Maxwell equations are solved for the given problem. /25

In order to simplify the analysis, let us introduce the following dimensionless parameters, except for ε:

$$\bar{b} = b\alpha, \quad (29)$$

$$\bar{\delta} = \delta\alpha, \quad (30)$$

$$\bar{x} = x\alpha, \quad (31)$$

$$\bar{\omega} = \mu_0 \sigma \omega s b^2. \quad (32)$$

The dependence of the induction components on the dimensionless coordinate $\bar{x}$ is shown in Figures 6 and 7. B_0 is the tangential induction component on the inductor surface, and ϕ is the angle at which the phase shifts between both components with respect to the phase of the tangential component on the inductor surface. The solid curves refer to the normal induction component; the broken curves refer to the tangential component. Graphs are given for different values of the parameter ε, and the value of $\varepsilon = 4.5$ corresponds to the force maximum for a given thickness $\bar{b} = 0.35$.

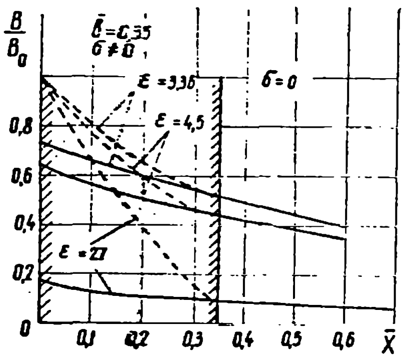

Figure 6

Damping of the Traveling Magnetic Field Amplitude in the Conductive Layer Above a One-Sided Inductor in the Case of $\bar{\delta} = 0$

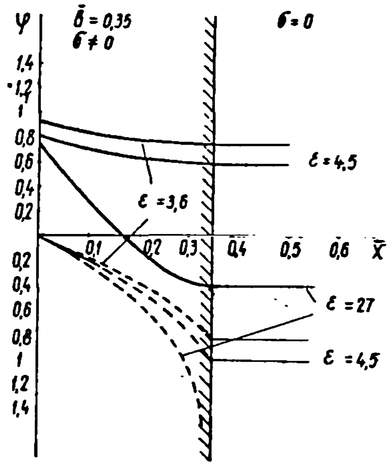

Figure 7

Change in the Phase of the Traveling Magnetic Field in a Conductive Layer Over a One-Sided Inductor in the Case of $\bar{\delta} = 0$

It may be seen from the graphs that both the amplitude and the argument ϕ of the normal induction component decrease as ε increases. The phase /26 of the normal induction component sharply changes based on the thickness of the metal layer during this surface effect. When passing through it, ϕ_x rapidly decreases, and the phase of the normal induction component lags behind the tangential component on the surface of the inductor.

In the case of $\mu = \infty$, H_0 equals the linear current load A_0. If the space is filled by a substance with $\sigma' \neq 0$ and $\mu' \neq \infty$ in the case of $x < 0$, then the following relationship holds between H_0 and A_0:

$$\frac{H_0}{A_0} = -\frac{2\mu'\alpha}{d}\,[(\alpha^2 \operatorname{sh}\alpha\delta + \beta^2 \operatorname{ch}\alpha\delta)\operatorname{sh}\beta b + \alpha\beta e^{\alpha\delta}\operatorname{ch}\beta b], \tag{33}$$

where

$$d = (\alpha+\beta)[\mu\beta'(\alpha\operatorname{ch}\alpha\delta + \beta\operatorname{sh}\alpha\delta) + \mu'\alpha(\alpha\operatorname{sh}\alpha\delta + \beta\operatorname{ch}\alpha\delta)]e^{\beta b} +$$
$$+ (\beta-\alpha)[\mu\beta'(\alpha\operatorname{ch}\alpha\delta - \beta\operatorname{sh}\alpha\delta) + \mu'\alpha(\alpha\operatorname{sh}\alpha\delta - \beta\operatorname{ch}\alpha\delta)]e^{-\beta b};$$
$$\beta' = \sqrt{\alpha^2 + \mu'\sigma'\omega s}.$$

The reaction of secondary currents in the conductive layer is characterized by a change in the vector of the normal induction component $\dot{B}_x/B_0$ on the inductor surface for $\delta = 0$, as a function of ε, and in the relative thickness of the conductive layer $\bar{b}$ (Figure 8). In this case ϕ characterizes

the difference between the angle at which the phase shifts between the normal induction component for different values of $\bar{b}$ and ε and the tangential component on the surface of the inductor. If $\varepsilon \to 0$, then $\left|\frac{\dot{B}_x}{B_0}\right| \to 1$ and $\phi \to \frac{\pi}{2}$.

Both the amplitude of the vector and ϕ decrease with an increase in ε. In the case of $\left|\frac{\dot{B}_x}{B_0}\right| \to 0$, the angle at which the phase shifts between both components strives to the limiting value $\frac{\pi}{4}$. Thus, for $\bar{b} \geqslant 1$ $\frac{\dot{B}_x}{B_0}$ strives to the limit from the left side of the bisector, and in the case of $\bar{b} = 0.1 - 0.2$ it strives to the limit from the right side. However, for $\bar{b} = 0.25 - 0.75$ the geometric location of the end of the vector has a point of inflection approaching the limit first from the right side, and then from the left side. In the case of $\bar{b} \to 0$, the geometric position of the end of the vector $\frac{\dot{B}_x}{B_0}$ represents a semicircle. In the case of $\bar{b} = 2$, the vector diagram practically coincides with the curve in the case of $\bar{b} = \infty$.

/27

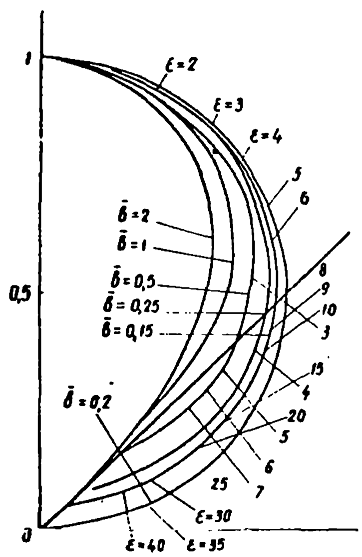

Figure 8

Geometric Location of the End of the Vector $\frac{\dot{B}_x}{B_0}$ as a Function of ε and $\bar{b}$

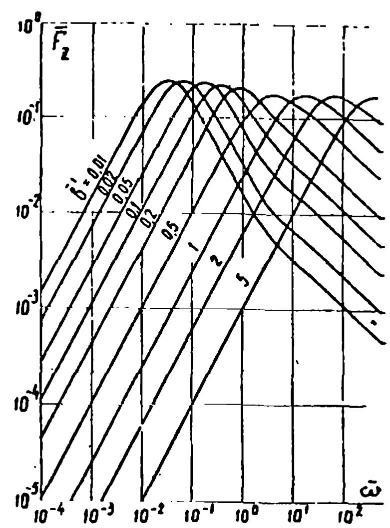

Figure 9

Dependence of $\bar{F}_z$ on $\bar{\omega}$ in the Case of $\bar{\delta} = 0$

Let us present the relationships for the components of the ponderomotive force in terms of the dimensionless criteria employed in the work (Ref. 10):

$$\bar{F}_z = \frac{F_{z\,ct}}{\mu H_0^2} = \frac{\bar{\omega}\bar{U}}{8\bar{b}^2\bar{R}'};$$

$$\bar{U} = \frac{\bar{b}\,\mathrm{sh}\,2\gamma_1}{\gamma_1} - \frac{\bar{b}\sin 2\gamma_2}{\gamma_2} + \frac{\gamma_1^2+\gamma_2^2}{\bar{b}}\left(\frac{\mathrm{sh}\,2\gamma_1}{\gamma_1} + \frac{\sin 2\gamma_2}{\gamma_2}\right) + 2(\mathrm{ch}\,2\gamma_1 - \cos 2\gamma_2);$$

$$\bar{R}' = (\mathrm{sh}^2\gamma_1 + \sin^2\gamma_2)\left[\frac{(\gamma_1^2+\gamma_2^2)^2}{\bar{b}^4}\mathrm{ch}^2\bar{\delta} + \mathrm{sh}\,2\bar{\delta} + \mathrm{sh}^2\bar{\delta}\right] + e^{2\bar{\delta}}\left[\frac{\gamma_1}{\bar{b}}\mathrm{sh}\,2\gamma_1 + \frac{\gamma_2}{\bar{b}}\sin 2\gamma_2 + \frac{\gamma_1^2+\gamma_2^2}{\bar{b}^2}(\mathrm{ch}^2\gamma_1 - \sin^2\gamma_2)\right] + \frac{2\gamma_1\gamma_2}{\bar{b}^3}\mathrm{ch}\,\bar{\delta}e^{\bar{\delta}}(\gamma_2\,\mathrm{sh}\,2\gamma_1 - \gamma_1\sin 2\gamma_2); \qquad (34)$$

$$\gamma_1 = \sqrt{\frac{\sqrt{\bar{\omega}^2+\bar{b}^4}+\bar{b}^2}{2}}; \quad \gamma_2 = \sqrt{\frac{\sqrt{\bar{\omega}^2+\bar{b}^4}-\bar{b}^2}{2}}.$$

The transverse component of the force has the following form /28

$$\bar{F}_x = \frac{F_{x\,ct}}{\mu H_0^2} = \frac{\bar{\omega}}{8\bar{b}^3\bar{R}'}\left[\frac{\bar{\omega}}{\bar{b}}(\mathrm{ch}\,2\gamma_1 - \cos 2\gamma_2) + 2(\gamma_2\,\mathrm{sh}\,2\gamma_1 - \gamma_1\sin 2\gamma_2)\right] \qquad (35)$$

where (F_{xct} and F_{zct} are the averaged transverse and longitudinal components of the force, respectively, acting upon a volume of substance 2b m^3 - i.e., on a column with unit area cut out of the plate .

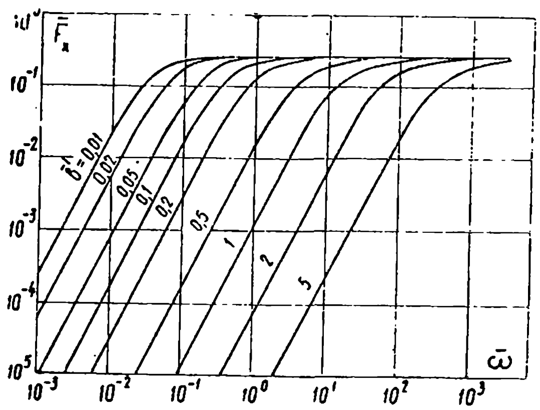

Figure 10

Dependence of $\bar{F}_x$ on $\bar{\omega}$ in the Case of $\bar{\delta} = 0$

Figures 9 and 10 present the dependences $\bar{F}_x$ and $\bar{F}_z$ according to (34) and (35) as a function of $\bar{\omega}$ for different values of the parameter $\bar{b}' = \frac{b}{\tau}$ in the case of $\delta = 0$.

The dependence of the ratio of the maximum values of the force components

$\frac{\bar{F}_{zmax}}{\bar{F}_{xmax}}$ on $\bar{b}$ and $\bar{\delta}$ is given in Figure 11. The limiting value of the force component F_x in the case of $\varepsilon \to \infty$ can be used as an approximate estimate of the maximum value of the function $F_z(\varepsilon)$. If the thickness of the conductive layer is small -- i.e., the depth to which the traveling magnetic field penetrates is larger than b -- and if the conductive layer is close to the inductor, then $F_{z\ max} \approx \lim_{\varepsilon\to\infty} F_x$.

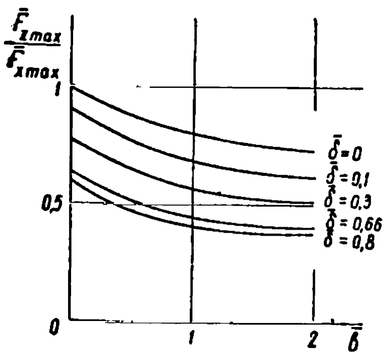

Figure 11

Dependence of $\frac{\bar{F}_{z\ max}}{\bar{F}_{x\ max}}$ on $\bar{b}$

The flux of electro-magnetic power through a unit of surface of the one-sided inductor also has two components $\dot{\Pi}_x$ and $\dot{\Pi}_z$: /29

$$\dot{\Pi}_x=\frac{1}{2} E_m^* \frac{B_{zm}}{\mu_0}; \quad \dot{\Pi}_z=-\frac{1}{2} E_x^* \frac{B_{xm}}{\mu_0}; \tag{36}$$

$$P_a=\mathrm{Re}\,\dot{\Pi}_x=\frac{\omega s B_0^2}{2\alpha\mu_0 R}\left\{\frac{\bar{\psi}_2}{2}(1+\bar{\psi}_1^2+\bar{\psi}_2^2)\,\mathrm{sh}\,2v_1+\right.$$
$$\left.+\frac{\bar{\psi}_1}{2}(\bar{\psi}_1^2+\bar{\psi}_2^2-1)\sin 2v_2+\bar{\psi}_1\bar{\psi}_2(\mathrm{ch}\,2v_1-\cos 2v_2)\right\}; \tag{37}$$

$$P_r=\mathrm{Im}\,\dot{\Pi}_x=\frac{\omega s B_0^2}{2\alpha\mu_0 R}\left\{\frac{\bar{\psi}_1(\bar{\psi}_1^2+\bar{\psi}_2^2+1)}{2}\,\mathrm{sh}\,2v_1-\right.$$
$$\left.-\frac{\bar{\psi}_2(\bar{\psi}_1^2+\bar{\psi}_2^2-1)}{2}\sin 2v_2+\bar{\psi}_1^2\,\mathrm{ch}\,2v_1+\bar{\psi}_2^2\cos 2v_2\right\}.$$

The tangential component of the Poynting vector has only a real part:

$$\Pi_z=\frac{\omega s B_0^2}{2\alpha\mu_0 R}[1+\bar{\psi}_1\,\mathrm{ch}\,2v_1+\bar{\psi}_2\sin 2v_2+$$
$$+(\bar{\psi}_1^2+\bar{\psi}_2^2+1)\,\mathrm{sh}^2 v_1+(\bar{\psi}_1^2+\bar{\psi}_2^2-1)\cos^2 v_2]. \tag{38}$$

Here

$$v_1=\alpha\bar{\psi}_1(\delta+b-x); \quad v_2=\alpha\bar{\psi}_2(\delta+b-x);$$
$$R=(\mathrm{sh}\,\bar{b}\bar{\psi}_1+\sin^2\bar{b}\bar{\psi}_2)[(\bar{\psi}_1^2+\bar{\psi}_2^2)^2\,\mathrm{ch}^2\bar{\delta}+\mathrm{sh}\,2\bar{\delta}+\mathrm{sh}^2\,\bar{\delta}]+$$
$$+e^{2\delta}[\bar{\psi}_1\,\mathrm{sh}\,2\bar{b}\bar{\psi}_1+\bar{\psi}_2\sin 2\bar{b}\bar{\psi}_2+(\bar{\psi}_1^2+\psi_2^2)\,(\mathrm{ch}^2\,\bar{b}\bar{\psi}_1-\sin^2\bar{b}\bar{\psi}_2)]+$$
$$+2\bar{\psi}_1\bar{\psi}_2\,\mathrm{ch}\,\bar{\delta}\,e^{\bar{\delta}}(\bar{\psi}_2\,\mathrm{sh}\,2\bar{b}\bar{\psi}_1-\bar{\psi}_1\sin 2\bar{b}\bar{\psi}_2).$$

Π_z expresses that portion of the electromagnetic energy which is propagated in the direction of motion of the traveling magnetic field.

The results derived from solving the problems described above may be applied in designing electromagnetic chutes for transporting molten ferrous metals. Similar problems were solved for electromagnetic mixing of a liquid metal in melting furnaces (Ref. 14 - 17, Ref. 20 - 22) and for non-contact flow meters for molten metal (Ref. 24, 25). For electromagnetic mixers, the problems were solved with many layers, with allowance for an insulation /30
layer above the inductor and the screen. Since the depth to which the traveling magnetic field penetrates in these cases is less than the thickness of the molten metal layer, it is usually assumed that its thickness is infinite. In view of the fact that the nature of electromagnetic processes in the electroconductive layer does not depend on other layers located between the layer under consideration and the inductor, the nature of these processes in a molten metal is the same as in conductive half-space.

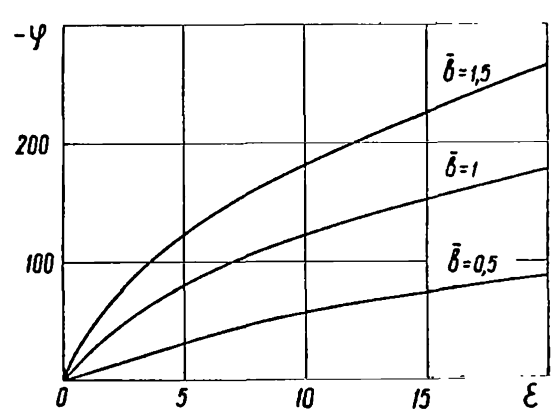

Figure 12

Dependence of the Phase Shift on ε

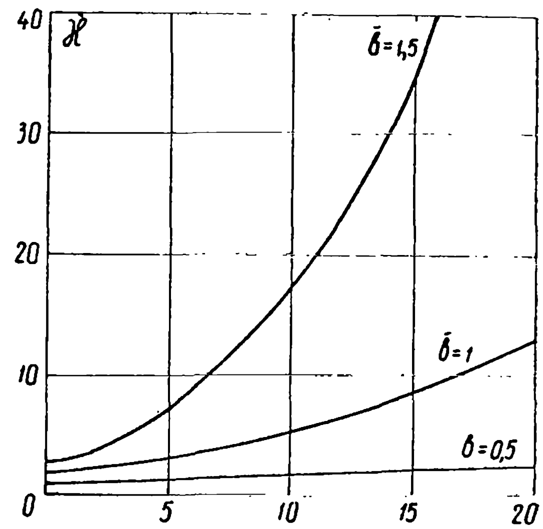

Figure 13

Dependence of the Ratio of the Magnetic Induction Amplitudes on ε

The problem was solved (Ref. 24) for a non-contact electroconductive medium velocity meter for a system consisting of an inductor, an electroconductive layer, and a ferromagnetic packet. It was assumed that $\mu = \infty$ for the inductor and the ferromagnetic packet /31

$$\varphi = -\operatorname{arctg}(\operatorname{th}\psi_1 b \operatorname{tg}\psi_2 b) \tag{39}$$

and

$$\varkappa = \sqrt{\frac{1}{2}(\operatorname{ch} 2\psi_1 b + \cos 2\psi_2 b}. \tag{40}$$

ϕ is the phase lag of the normal component of the traveling magnetic field when it passes through a layer of the electroconductive medium, and $\varkappa$ is the ratio of the amplitudes of the normal induction component.

Figures 12 and 13 present the dependence of the angle at which the phase shifts ϕ and the ratio of the amplitudes upon the parameters ε for the three values of $\bar{b}$ = bα.

4. Electromagnetic Processes in the Traveling Magnetic Field of a Two-Sided Inductor

Several authors have investigated the problems related to the phenomena in the field of a two-sided inductor. The problem was solved in the works (Ref. 9, 10) for any value of ϕ in a general form. These problems are of interest in designing equipment which employs the normal component of the electromagnetic force, and for equipment operating in the case of ϕ = 180°. They are also of interest for determining the influence upon the tangential force component when ϕ deviates from 0.

The problem was solved in (Ref. 10) for one layer (the tangential component of the magnetic field was defined on the layer surface). In this case, the following values of the fields hold within the layer:

$$H_x=\frac{i\alpha H_0}{\beta\,\mathrm{sh}\,2\beta b}[\mathrm{ch}\,\beta(b+x)+e^{i\varphi}\,\mathrm{ch}\,\beta(b-x)]\,e^{i(\omega st-\alpha z)}, \tag{41}$$

$$H_z=\frac{H_0}{\mathrm{sh}\,2\beta b}[\mathrm{sh}\,\beta(b+x)-e^{i\varphi}\,\mathrm{sh}\,\beta(b-x)]\,e^{i(\omega st-\alpha z)}, \tag{42}$$

$$E_\varphi=-\frac{i\omega\mu H_0}{\beta\,\mathrm{sh}\,2\beta b}[\mathrm{ch}\,\beta(b+x)+e^{i\varphi}\,\mathrm{ch}\,\beta(b-x)]\,e^{i(\omega st-\alpha z)}. \tag{43}$$

In this case, the thickness of the conductive layer equals 2b.

Relationships (41) and (42) may be employed to obtain the equation for the force lines of the magnetic field, which has the following form for no-load operation: /32

$$\frac{dx}{dz}=\frac{\mathrm{ch}\,\alpha(b+x)\sin\alpha z+\mathrm{ch}\,\alpha(b-x)\sin(\alpha z-\psi)}{\mathrm{sh}\,\alpha(b+x)\cos\alpha z-\mathrm{sh}\,\alpha(b-x)\cos(\alpha z-\psi)}. \tag{44}$$

In contrast to the angle ϕ which was introduced previously, here Ψ characterizes the phase shift between currents flowing in the direction of the y-axis, since it is more advantageous to measure the current instead of the magnetic field in practice. Only in two cases does $\phi = \Psi$ hold: when they both equal 0, or 180°. Figures 14 and 15 present a picture of the force lines of the magnetic field during no-load operation (calculated by Yu. K. Krumin) for values of Ψ equalling 0 and 90°, in the case of $\frac{2b}{\tau} = 0.5$ - i.e., when the thickness of the plate is two times smaller than τ, since the dependence on Ψ is more apparent for thin plates than it is for thick plates.

In this case, it was found that two components (which are different from zero) of the ponderomotive force which is averaged over time and space may exist - the longitudinal component which is in operation in the direction of motion of the field, and the transverse component which operates in /33 the direction of the x-axis.

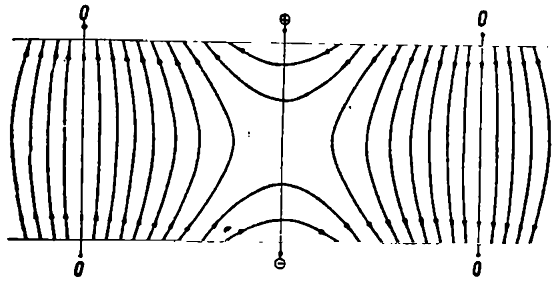

Figure 14

Force Lines of the Traveling Magnetic Field in the Clearance of a Two-Sided Inductor in the Case of $\Psi = 0$

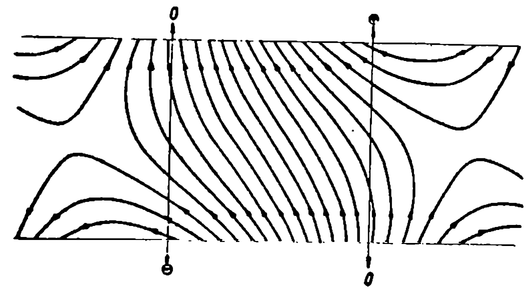

Figure 15

Force Lines of the Traveling Magnetic Field in the Clearance of a Two-Sided Inductor in the Case of $\Psi = 90°$

The force components may be expressed as follows in dimensionless form:

$$\overline{F}_z = \frac{F_{z\,ct}}{\mu H_0^2} = \frac{1}{2}[(1+\cos\varphi)\overline{F}_0 + (1-\cos\varphi)\overline{F}_{180}], \tag{45}$$

where

$$\overline{F}_0 = \overline{b}\,\frac{\gamma_2\,\mathrm{sh}\,2\gamma_1 + \gamma_1 \sin 2\gamma_2}{(\gamma_1^2+\gamma_2^2)(\mathrm{ch}\,2\gamma_1 - \cos 2\gamma_2)}; \tag{46}$$

$$\overline{F}_{180} = \overline{b}\,\frac{\gamma_2\,\mathrm{sh}\,2\gamma_1 - \gamma_1 \sin 2\gamma_2}{(\gamma_1^2+\gamma_2^2)(\mathrm{ch}\,2\gamma_1 + \cos 2\gamma_2)}; \tag{47}$$

$$\overline{F}_x = \frac{F_{x\,ct}}{\mu H_0^2} = \sin\varphi\,\frac{(\gamma_1^2-\gamma_2^2)\,\mathrm{sh}\,2\gamma_1 \sin 2\gamma_2}{(\gamma_1^2+\gamma_2^2)(\mathrm{sh}^2 2\gamma_1 + \sin^2 2\gamma_2)}; \tag{48}$$

where F_{xct} and F_{zct} designate the averaged transverse and longitudinal force components, respectively, acting upon a column cut out of a plate with a cross section of 1 unit of area.

Figures 16-18 graphically illustrate the dependence of (46) - (48) on $\overline{\omega}$, but, instead of $\overline{b}$, a more convenient criterion is introduced

$$b' = \frac{2b}{\tau}.$$

In the case of $\overline{\omega} \ll \overline{b}$, a directly proportional relationship always exists between the magnitude of the force and the value of the dimensionless criterion $\overline{\omega}$.

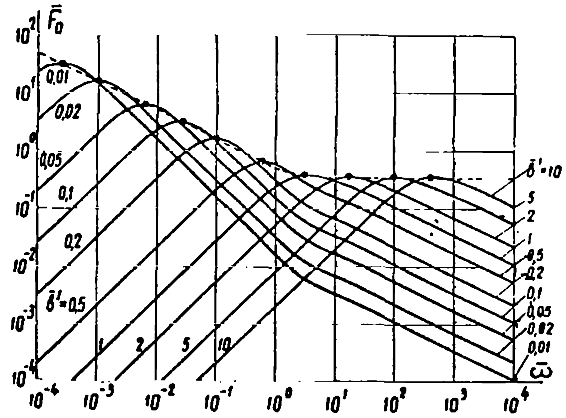

Figure 16

Dependence of $\bar{F}_0$ on $\bar{\omega}$

In the case of $\bar{\omega} \gg \bar{b}$ and $\bar{\omega} \gg 1$ - i.e., when there is a strongly expressed surface effect - $\bar{F}_0 \approx \bar{F}_{180} \sim \frac{1}{\sqrt{2\bar{\omega}}}$, where the z-component of force strives to zero in inverse proportion to $\sqrt{2\bar{\omega}}$. For the z-force component, when the inductor is switched on concurrently with $\bar{F}_0$ there is one characteristic region -- in the case of $\bar{\omega} \gg \bar{b}$ - when $\phi = 0°$, but in the case of $\bar{\omega} \ll 1$ the force changes in inverse proportion to the first power of $\bar{\omega}$. For $\bar{F}_{180}$, all the maxima of the curves occur in the case of $\bar{\omega} \gg 2.5$. There is no such limit for $\bar{F}_0$, and the maximum value of the force increases with a decrease in $\bar{b}$, shifting toward smaller values of $\bar{\omega}$ simultaneously. When there is a sharply expressed skin effect, the values of $\bar{F}_0$ and $\bar{F}_{180}$ coincide, and the maxima strive to the value of 0.354. /34 /35

The transverse force component $\bar{F}_x$ also has maxima with respect to $\bar{\omega}$. When there is a decrease in $\bar{b}$, the maximum value of $\frac{\bar{F}_x}{\sin \phi}$ strives to the value of 0.25. In contrast to $\bar{F}_z$, $\bar{F}_x$ is an alternating function of the condition $\bar{\omega}$. However, the absolute values of its extremums decrease so rapidly that only the region of the first maximum is of practical importance. In the case of $\bar{\omega} \gg \bar{b}$, we have

$$\bar{F}_x \approx \sin\varphi \frac{\bar{b}^2 \sin\sqrt{2\bar{\omega}}}{\bar{\omega}\,\mathrm{sh}\sqrt{2\bar{\omega}}}. \qquad (49)$$

If the surface currents are given on the surface of the inductor

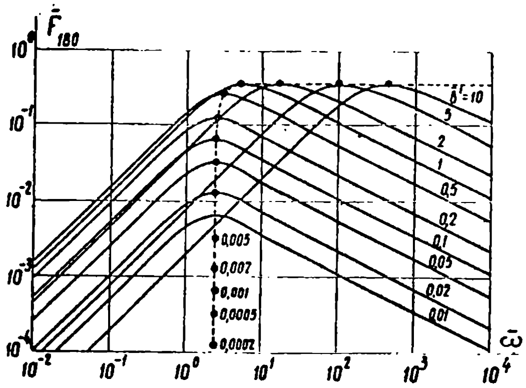

Figure 17

Dependence of $\bar{F}_{180}$ on $\bar{\omega}$

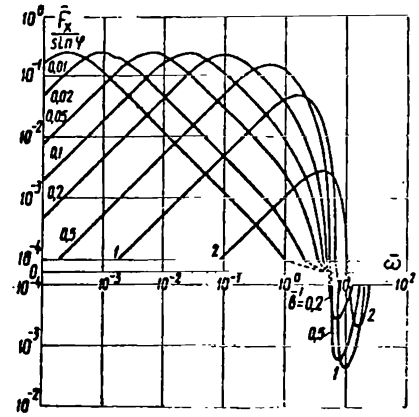

Figure 18

Dependence of $\dfrac{\bar{F}_x}{\sin\phi}$ on $\bar{\omega}$

$$A = A_0\, e^{i(\omega st - \alpha z)} \qquad \text{for} \quad x = +b;$$
$$A = A_0\, e^{i(\omega st - \alpha z + \Psi)} \qquad \text{for} \quad x = -b$$

as well as the space behind the surface currents - i.e., if it is occupied by a substance with the specific conductivity σ' and the magnetic permeability μ' in the case of $|x|>b$ - then the field strength $H_0 = H_{z0}$ in the case of $x = b$ and A_0 are related by the following relationship :

$$\frac{H_0}{A_0} = \frac{\mu'\beta\left[(\mu'\beta+\mu\beta')e^{2\beta b} - (\mu'\beta-\mu\beta')e^{-2\beta b} - 2\mu\beta'\right]e^{i\varphi}}{(\mu'\beta+\mu\beta')^2 e^{2\beta b} - (\mu'\beta-\mu\beta')^2 e^{-2\beta b}} \tag{50}$$

(μ is the magnetic permeability of the conductive band).

The angles at which the phase shifts between the fields ϕ and the currents Ψ in the inductor are related as follows:

$$\varphi = \operatorname{arctg}\frac{\operatorname{Im}\dfrac{[(\mu'\beta+\mu\beta')e^{2\beta b} - (\mu'\beta-\mu\beta')e^{-2\beta b}]\,e^{i\psi} - 2\mu\beta'}{[(\mu'\beta+\mu\beta')e^{2\beta b} - (\mu'\beta-\mu\beta')e^{-2\beta b}] - 2\mu\beta' e^{i\psi}}}{\operatorname{Re}\dfrac{[(\mu'\beta+\mu\beta')e^{2\beta b} - (\mu'\beta-\mu\beta')e^{-2\beta b}]\,e^{i\psi} - 2\mu\beta'}{[(\mu'\beta+\mu\beta')e^{2\beta b} - (\mu'\beta-\mu\beta')e^{-2\beta b}] - 2\mu\beta' e^{i\psi}}}. \tag{51}$$

In the case of $\Psi = 0°$, $\phi = \Psi$ and

$$\frac{H_0}{A_0} = \frac{1}{1+\frac{\bar{\mu}\beta'}{\bar{\mu}'\beta}\operatorname{cth}\beta b}. \quad (52)$$

In the case of $\Psi = 180°$ $\phi = \Psi$ and /36

$$\frac{H_0}{A_0} = \frac{1}{1+\frac{\bar{\mu}\beta'}{\bar{\mu}'\beta}\operatorname{th}\beta b}, \quad (53)$$

where

$$\bar{\mu} = \frac{\mu}{\mu_0}; \quad \bar{\mu}' = \frac{\mu'}{\mu_0}.$$

I. A. Tyutin and E. K. Yankop (Ref. 6) have solved the problem for $\phi = 0°$ and the number of layers n = 5, taking into account both the influence of the immobile channel walls, and the non-conductive heat-insulating layers. Then $s_1 = s_2 = s_4 = s_5 = 1$;

$$\sigma_1 = \sigma_5 = 0; \qquad \sigma_2 = \sigma_4 = \sigma_t; \qquad \sigma_3 = \sigma;$$

$$\beta_1 = \beta_5 = \alpha; \qquad \beta_4 = \beta_2 = \beta_t; \qquad \beta_3 = \beta;$$

$$b_2 = b_4 = b_t; \qquad b_3 = b; \qquad b_1 = b_5 = b_i;$$

$$\varphi = 0°; \qquad H_{01} = H_{02} = H_0; \qquad \mu_{ind} = \infty.$$

The normal component of magnetic induction on the inductor surface is assumed to be given.

R. A. Petrovich solved a similar problem, defining the tangential component on the inductor surface. Since it is more advantageous to define the tangential component, let us only investigate the solution of Petrovich. On the other hand, if we know one solution, we may readily obtain another solution by means of the following relationship

$$H_{x0} = iH_0 \operatorname{cth} \alpha b_t, \quad (54)$$

where H_{x0} is the amplitude of the normal component of the magnetic field on the inductor surface.

Let us write the amplitudes of the induction components in separate regions of the clearance:

$$B_{xV} = \frac{\mu_0 A_0 i}{R}(F \operatorname{ch} \alpha x - L \operatorname{sh} \alpha x), \quad (55)$$

$$B_{xIV} = \frac{\mu_0 A_0 i\alpha}{R}\left[\beta \operatorname{sh} \beta \frac{b}{2} \operatorname{sh} \beta_t \left(\frac{b}{2} - x\right) + \beta_t \operatorname{ch} \beta \frac{b}{2} \operatorname{ch} \beta_t \left(\frac{b}{2} - x\right)\right], \quad (56)$$

$$B_{xIII} = \frac{\mu_0 A_0 \beta_t \alpha i}{R} \operatorname{ch} \beta x, \quad (57)$$

$$B_{zV} = \frac{\mu_0 A_0}{R} [F \operatorname{sh} \alpha x - L \operatorname{ch} \alpha x], \tag{58}$$

$$B_{zIV} = \frac{\mu_0 A_0 \beta_t}{R} \left[\beta \operatorname{sh} \beta \frac{b}{2} \operatorname{ch} \left(\beta_t x - \beta_t \frac{b}{2} \right) + \right.$$

$$\left. + \beta_t \operatorname{ch} \beta \frac{b}{2} \operatorname{sh} \beta_t \left(x - \frac{b}{2} \right) \right], \tag{59}$$

$$B_{zIII} = \frac{\mu_0 A_0 \beta \beta_t}{R} \operatorname{sh} \beta_t x. \tag{60}$$

The density of the induced currents in regions III and IV is:

$$j_{III} = -\frac{\mu_0 A_0 \omega si}{R} \beta_t \operatorname{ch} \beta x, \tag{61}$$

$$j_{IV} = -\frac{\mu_0 A_0 i \omega}{R} \left[\beta \operatorname{sh} \beta \frac{b}{2} \operatorname{sh} \beta_t \left(x - \frac{b}{2} \right) + \right.$$

$$\left. + \beta_t \operatorname{ch} \beta \frac{b}{2} \operatorname{ch} \beta_t \left(x - \frac{b}{2} \right) \right]. \tag{62}$$

Here

$$R = \alpha\beta_t \operatorname{sh} \alpha b_i \operatorname{ch} \beta_t b_t \operatorname{ch} \beta \frac{b}{2} + \beta_t^2 \operatorname{ch} \alpha b_i \operatorname{sh} \beta_t b_t \operatorname{ch} \beta \frac{b}{2} +$$

$$+ \beta\beta_t \operatorname{ch} \alpha b_i \operatorname{ch} \beta_t b_t \operatorname{sh} \beta \frac{b}{2} + \alpha\beta \operatorname{sh} \alpha b_i \operatorname{sh} \beta_t b_t \operatorname{sh} \beta \frac{b}{2}; \tag{63}$$

$$F = \frac{1}{4} \left[(\alpha + \beta_t)(\beta_t + \beta) \operatorname{ch} (\beta \frac{b}{2} + \beta_t b_t - \alpha \frac{b}{2} - \alpha b_t) + \right.$$

$$+ (\alpha - \beta_t)(\beta + \beta_t) \operatorname{ch} (\beta \frac{b}{2} + \beta_t b_t + \alpha \quad + \alpha b_t) +$$

$$+ (\alpha - \beta_t)(\beta_t - \beta) \operatorname{ch} (-\beta \frac{b}{2} + \beta_t b_t + \alpha \frac{b}{2} + \alpha b_t) + \tag{64}$$

$$\left. + (\alpha + \beta_t)(\beta_t - \beta) \operatorname{ch} (-\beta \frac{b}{2} + \beta_t b_t - \alpha \frac{b}{2} - \alpha b_t) \right];$$

$$L = \frac{1}{4} \left[(\alpha + \beta_t)(\beta + \beta_t) \operatorname{sh} (\alpha \frac{b}{2} + \alpha b_t - \beta \frac{b}{2} - \beta_t b_t) + \right.$$

$$+ (\alpha - \beta_t)(\beta + \beta_t) \operatorname{sh} (\beta \frac{b}{2} + \beta_t b_t + \alpha \frac{b}{2} + \alpha b_t) +$$

$$+(\alpha+\beta_t)(\beta_t-\beta)\operatorname{sh}\left(\beta\frac{b}{2}-\beta_t b_t+\alpha\frac{b}{2}+\alpha b_t\right)+$$

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$$+(\alpha-\beta_t)(\beta_t-\beta)\operatorname{sh}\left(\beta_t b_t-\beta\frac{b}{2}+\alpha\frac{b}{2}+\alpha b_t\right)\Big]. \qquad (65)$$

The electromagnetic pressure acting upon the conductive band, which is averaged over time and over layer thickness and which is referred to a unit of layer length, equals

$$p_e=\frac{\mu_0^2 A_0^2\sigma\alpha\omega s}{4b}\left|\frac{\beta_t}{R}\right|^2\left(\frac{\operatorname{sh}\psi_1 b}{\psi_1}+\frac{\sin\psi_2 b}{\psi_2}\right). \qquad (66)$$

The normal component of the Poynting vector on the surface $x = \frac{\delta}{2}$ has the following form

$$\dot{\Pi}_{xt}=\frac{1}{2}E_{yt}H_{zt}^{*}=$$
$$=\frac{i\omega\mu_0^2 A_0^2}{\alpha R^2}(F^*\operatorname{ch}\alpha x-L^*\operatorname{sh}\alpha x)(F\operatorname{sh}\alpha x-L\operatorname{ch}\alpha x). \qquad (67)$$

By computing the active power, we may find the power which is liberated in molten metal, and we may determine the losses at the channel walls in the form of the difference between the total flux of active power passing through the channel wall and the power liberated in them.

Let us investigate certain simplified cases (Ref. 7).

If $\sigma_t = 0$, the electromagnetic pressure of the pump will be

$$p_e=\frac{\mu_0 A_0^2 l F_1 F_2}{b(K_1^2+K_2^2)}, \qquad (68)$$

where $\ell = 2\tau p_n$ is the length of the pump active zone; p_n is the number of pairs of winding poles;

$$K_1=F_1\operatorname{sh}\frac{1}{2}\alpha(\delta-b)+F_3\operatorname{ch}\frac{1}{2}\alpha(\delta-b);$$

$$K_2=F_2\operatorname{ch}\frac{1}{2}\alpha(\delta-b);$$

$$F_1=\operatorname{ch}\psi_1 b+\cos\psi_2 b;$$

$$F_2=\frac{\psi_1}{\alpha}\sin\psi_2 b+\frac{\psi_2}{2}\operatorname{sh}\psi_1 b;$$

$$F_3=\frac{\psi_1}{\alpha}\operatorname{sh}\psi_1 b-\frac{\psi_2}{\alpha}\sin\psi_2 b.$$ /39

The active power transmitted to the molten metal and the electromagnetic pressure are related by the following simple relationship

$$P_a=p_e Q_s, \tag{69}$$

where Q_s is the output of the pump corresponding to the synchronous velocity of the molten metal

$$Q_s=2\tau f ab \tag{70}$$

(where a is the width of the pump channel in the direction of the y-axis).

The electromagnetic power P_a represents the sum of two components: the power of the Joule heat losses

$$P_b=p_e s Q_s \tag{71}$$

and the mechanical power

$$P_2=p_e(1-s)Q_s=p_e Q, \tag{72}$$

where Q is the real output of the pump.

The reactive power consumed when the magnetic flux passes through the clearance is

$$P_r=\frac{\mu_0 A_0^2 2\tau f al}{K_1^2+K_2^2}\left[F_1 F_3 \operatorname{ch}\alpha(\delta-b)+\frac{1}{2}\left(F_1^2+F_4\sqrt{1+\varepsilon^2}\right)\operatorname{sh}\alpha(\delta-b)\right]. \tag{73}$$

Let us examine the special case $\varepsilon \ll 1$. In physical terms, this means that the magnetic induction produced by the induction currents in the conductive band is considerably less than the primary (exciting) induction. The expressions for the electromagnetic pressure and powers may then be considerably simplified: /40

$$p_e=\frac{\mu_0 A_0^2 \alpha l k_\tau}{2\operatorname{sh}^2\frac{\alpha b}{2}}\varepsilon, \tag{74}$$

$$P_a=\frac{\mu_0 A_0^2 \alpha l k_\tau ab2\tau f}{2\operatorname{sh}^2\frac{\alpha b}{2}}\varepsilon, \tag{75}$$

$$P_r=2\tau f\mu_0 A_0^2 al\operatorname{cth}\frac{\alpha\delta}{2}. \tag{76}$$

Here we have

$$k_\tau = \frac{1}{2} \cdot \frac{\alpha b + \operatorname{sh} \alpha b}{\alpha b}. \tag{77}$$

As may be seen from expressions (74 and 75), the electromagnetic pressure and the active power are directly proportional to ε in the case of $\varepsilon \ll 1$.

If $\tau \gg b$, then the tangential component of the magnetic induction is small as compared with the normal component. The latter is almost uniform with respect to the clearance height.

In this case, we have

$$p_e = \frac{2\mu_0 l \varepsilon A_0^2}{\alpha(\delta^2 + \varepsilon^2 b^2)}, \tag{78}$$

$$P_a = \frac{2\mu_0 A_0^2 2\tau f a b l \varepsilon}{\alpha(\delta^2 + \varepsilon^2 b^2)}, \tag{79}$$

$$P_r = \frac{\mu_0 A_0^2 2\tau f a l b s}{\alpha(\delta^2 + \varepsilon^2 b^2)} \left\{ 2\varepsilon - \frac{1}{2} [4 + (1 + \varepsilon^2 + \varepsilon\sqrt{1+\varepsilon^2}) \alpha^2 b (\delta - b)] \right\}. \tag{80}$$

The studies (Ref. 18, 19) solved the problems with respect to electromagnetic processes in a conductive layer having the finite width 2a (i.e., with allowance for the transverse edge effect). It is assumed that the magnetic field in the clearance of the inductor was plane-parallel (see the article by Yu. A. Mikel'son in the present collection). In particular, this work obtained the expressions for the electromagnetic pressure p_c and the power P_a. Assuming that a = ∞ in these formulas, i.e., assuming that the width of the plate was infinitely large, they were transformed into

$$p = \frac{1}{1+\varepsilon'^2} \tau l f \sigma B^2 \tag{81}$$

and /41

$$P_a = \frac{1}{1+\varepsilon'^2} 4 a b l \tau \sigma f B^2, \tag{82}$$

i.e., they correspond to the approximation of the solution of R. A. Petrovich in the case of $\tau \gg \delta$ given above. Here $\varepsilon' = \varepsilon \frac{b}{\delta}$; B is the induction in the clearance during no-load operation.

The theory for electromagnetic processes in an infinitely wide layer, located in a traveling magnetic field, has been developed in greater detail than in layers of finite width, since a mathematical analysis is simpler.

REFERENCES

1. Ollendorff, F. Einheitliche Theorie der Drehfeldmaschinen (Unified Theory of Rotating Field Machines). Archive f. Elektrotechnik, 24, 2, 1930.

2. Lopukhina, Ye.M. Issledovaniye asinkhronnogo dvigatelya s rotorom v vide pologo tsilindra (Study of the Asynchronous Engine with a Rotor in the Form of a Complete Cylinder). Elektrichestvo 5, 26, 1950.
3. Tyutin, I.A. Mekhanicheskiye sily v begushchem magnitnom pole (Mechanical Forces in a Traveling Magnetic Field). V kn: Voprosy energetiki (In the Book: Problems of Energetics), 3. Izdatel'stvo AN Latv. SSR, 1955.
4. Tyutin, I.A. Vvedeniye v teoriyu induktsionnykh nasosov (Introduction to the Theory of Induction Pumps). Trudy Instituta Fiziki, AN Latv. SSR, 8, 1956.
5. Tyutin, I.A. Elektromagnitnyye nasosy dlya zhidkikh metallov (Electromagnetic Pumps for Molten Metals). Riga, 1959.
6. Tyutin, I.A., Yankop, E.K. Elektromagnitnyye protsessy v induktsionnykh nasosakh dlya zhidkikh metallov (Electromagnetic Processes in Induction Pumps for Molten Metals). Trudy Instituta Fiziki AN Latv. SSR, 8, 1956.
7. Liyelpeter, Ya.Ya., Petrovich, R.A. K teorii ploskikh induktsionnykh nasosov (Theory of Plane Induction Pumps). Izvestiya AN Latv. SSR, Seriya Fizicheskikh i Tekhnicheskikh Nauk, 1, 1964.
8. Liyelpeter, Ya.Ya., Tyutin, I.A. Metodika rascheta induktsionnykh nasosov dlya zhidkogo metalla (Method for Designing Induction Pumps for Molten Metal). Trudy Instituta Fiziki AN Latv. SSR, 8, 1956.
9. Mikel'son, Yu.A. Provodyashchiy sloy v begushchem elektromagnitnom pole dvukhstoronnego nesimmetrichnogo induktora (Conductive Layer in a Traveling Electromagnetic Field of a Two-Sided Non-symmetrical Inductor). Izvestiya AN Latv. SSR, Seriya Fizich. i Tekhnicheskikh Nauk, 2, 1965.
10. Veze, A.K., Krumin', Yu.K. Ob elektromagnitnoy sile, deystvuyushchey na beskonechno shirokiy provodyashchiy sloy v begushchem magnitnom pole ploskikh induktorov (Electromagnetic Force Acting Upon an Infinitely Wide Conductive Layer in a Traveling Magnetic Field of Plane Inductors). Magnitnaya Gidrodinamika, 4, 1965.
11. Kirko, I.M. Kriterii podobiya elektrodinamicheskikh yavleniy pri otnositel'nom dvizhenii magnitnogo polya i provodyashchey sredy (Criteria for Similarity in Elektrodynamic Phenomena During the Relative Motion of a Magnetic Field and a Conductive Medium). V kn: Voprosy energetiki (In the Book: Problems of Energetics), 3, 1955.
12. Veze, A.K., Liyelausis, O.A., Petrovich, R.A., Ulmanis, L.A. Provodyashiy sloy v begushchem elektromagnitnom pole odnostoronnego induktora (Conductive Layer in a Traveling Electromagnetic Field of a One-Sided Inductor). V kn: Voprosy magnitnoy gidrodinamiki (In the Book: Problems of Hydrodynamics), III. Izdatel'stvo AN Latv. SSR, 1963.
13. Veze, A.K., Ulmanis, L.Ya. Raspredeleniye elektromagnitnogo polya i ponderomotornykh sil v beskonechnoy provodyashchey polose, pomeshchennoy v begushchem magnitnom polc odnostoronnego induktora (Distribution of the Electromagnetic Field and Ponderomotive Forces in an Infinitely Conductive Band Placed in a Traveling Magnetic Field of a One-sided Inductor). In press.
14. Vaynberg, G.S. K teorii ustroystva dlya elektromagnitnogo peremeshivaniya rasplavlennogo metalla v dugovykh elektropechakh (Theory of Equipment for the Electromagnetic Mixing of Molten Metal in Arc Electric Furnaces). Elektrichestvo, 2, 1958.

15. Vaynberg, G.S. O vybore chastoty ustroystv dlya elektromagnitnogo per /42 emeshivaniya metallov v elektropechi (Selection of Equipment Frequency for Electromagnetic Mixing of Metals in an Electric Furnace). Elektrichestvo 5, 1958.
16. Kochnev, E.K. K teorii ustroystv dlya elektromagnitnogo peremeshivaniya rasplavlennogo metalls (Theory of Equipment for Electromagnetic Mixing of Molten Metal). Elektrichestvo, 7, 1959.
17. Kochnev, E.K., Rezin, M.G. Issledovaniye ustroystva po elektromagnitnomu peremeshivaniyu rasplavlennogo metalla (Study of Equipment for Electromagnetic Mixing of Molten Metal). Izvestiya Vysshikh Uchebnykh Zavedeniy, Elektrotekhnika, 9, 1962.
18. Vol'dek, A.I. Toki i usiliya v sloye metalla ploskikh induktsionnykh nasosov (Currents and Stresses in the Layer of Metal of Plane Induction Pumps). Izvestiya Vysshikh Uchebnykh Zavedeniy, Elektromekhanika, 1,1959.
19. Yanes, Kh. I. Uchet vliyaniya vtorichnoy sistemy v lineynoy ploskoy magnitogidrodinamicheskoy mashine (Influence of the Secondary System in the Linear Plane Magnetohydrodynamic Machine). Trudy Tallinskogo Politekhn Instituta, Seriya A, 197, 1962.
20. Vortnichuk, N.I., Krutyanskiy, M.M. O vybore optimal'noy chastoty toka statora pri peremeshivanii zhidkoy stali s pomoshch'yu begushchego magnitnogo polya (Selection of the Optimum Frequency of the Stator Current When Molten Steel is Mixed by Means of a Traveling Magnetic Field). V kn: Voprosy magnitnoy gidrodinamiki i dinamiki plazmy (Problems of Magnetic Hydrodynamics and Plazma Dynamics). Izdatel'stvo AN Latv. SSR,1959.
21. Ostroumov, G.A. O peremeshivanii rasplavlennykh metallov begushchim magnitnym polem (Mixing of Molten Metals by Means of a Traveling Magnetic Field). V kn: Voprosy magnitnoy gidrodinamiki i dinamiki plazmy (Problems of Magnetic Hydrodynamics and Plazma Dynamics). Izdatel'stvo AN Latv. SSR, 1959.
22. Ostroumov, G.A. Fiziko-Matematicheskiye osnovy magnitnogo peremeshivaniya rasplavov (Physico-Mathematical Bases of Magnetic Mixing of Melts.). Metallurgizdat, 1961.
23. Schilder, J. Pohybujíci se elektromagnetické pole ve vodivém prostredí (Electromagnetic Field Motion in a Conductive Medium). Elektrotechnický Obzor, 3, 1959.
24. Ulmanis, L.Ya. Beskontaktnyy raskhodomer dlya zhidkikh metallov (Non-Contact Flow Meter for Molten Metals). Byulleten' Izobreteniy, 19, 1962.
25. Ul'manis, L.Ya. Fizicheskiye yavleniya pri induktsionnom vozdeystvii begushchego magnitnogo polya na sloy zhidkogo metalla (Physical Phenomena During Induction Interaction of a Traveling Magnetic Field on a Layer of Liquid Metal). Avtoref. Kanddiss. (Author's Abstract of Candidate's Dissertation). Riga, 1963.
26. Ul'manis, L.Ya. K voprosu o krayevykh effektakh v lineynykh induktsionnykh (Problems of Edge Effects in Linear Induction Pumps). Trudy Instituta Nasosakh Fiziki, AN Latv. SSR, 8, 1956.
27. Watt, D.A. A Study in Design of Traveling Field Electromagnetic Pumps for Liquid Metals. Harwell, 1955.
28. Valdmanis, Ya.Ya. Elektromagnitnyye sily deystvuyushchiye na beskonechnuyu provodyashchuyu polosu v pöle odnostoronnego induktora trekhfaznogo toka (Electromagnetic Forces Acting Upon an Infinite Conductive Band in the Field of a One-Sided Inductor of Tri-Phase Current.) Izvestiya AN Latv. SSR, Seriya Fizich. i Tekhnicheskikh Nauk, 1, 1965.

HIGHER SPATIAL HARMONICS OF THE MAGNETIC FIELD OF AN INDUCTION MHD MACHINE /43

Yu. Ya. Mikel'son

1. Introduction

In terms of their principle of operation, induction MHD machines are similar to asynchronous electric machines. Just as in asynchronous engines, the winding of the stator produces a traveling magnetic field, under the influence of which currents are induced in the molten metal of the MHD machine (in the rotor of the asynchronous machine). The interaction of these currents with the magnetic field of the stator leads to the formation of ponderomotive forces. Therefore, there is a certain similarity between certain problems involved in the theory of asynchronous engines and induction MHD machines. One of these problems is the deviation of the traveling magnetic field from a sinusoidal one. This deviation is caused by several factors. The most important factor is the spatial distribution of the stator winding and the nonuniformity of the air gap (projections and grooves on the steel surface of the stator). These factors occur both in asynchronous engines and in MHD machines. However, they may have a different influence upon the operation of the machine due to differences in both devices. We would like to emphasize the following differences:

(1) The "rotor" of the MHD machine represents a solid, liquid conductive medium, in contrast to the discrete conductors of the rotor winding of the asynchronous machine;

(2) The clearance between the rotor and the stator of the asynchronous machine may differ greatly from this clearance in the induction MHD machine;

(3) The stator of the induction MHD machine may be both one-sided and two-sided;

(4) The operational regimes of the machines under consideration are different. As a rule, the asynchronous engines operate with slipping which is close to zero. Not all of these differences play the same role. An increase in the clearance between the rotor and the stator in a MHD machine, as compared with an asynchronous machine, reduces the role of the higher spatial harmonics (h.s.h.), while the wide region of slippings and the solid "rotor" of /44 the MHD machine may lead to an increase in the influence of the h.s.h. upon it.

Many authors have investigated the harmonics produced due to the spatial distribution of the multiphase stator winding, and have determined their influence upon the motion of the rotor. The results derived from a large number of these works have been generalized in (Ref. 1).

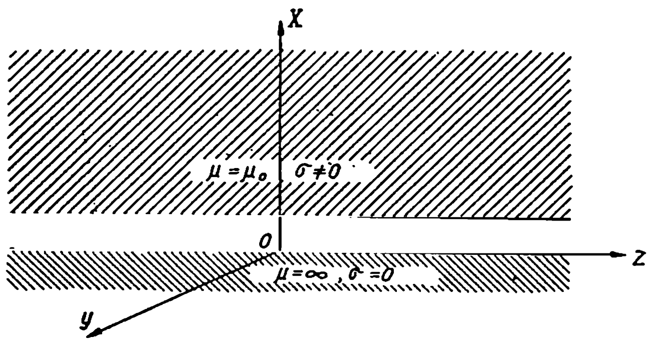

Figure 1

Infinite Layer of a Conductor (Above) and a Magnetic Current (Below). The Linear Current Load (Ref. 12) is Given on the Surface of the Magnetic Circuit.

The spatial distribution of a winding of multiphase current leads to the fact that a magnetic field is produced which consists of an infinite series of harmonics traveling in the opposite directions. In particular, harmonics of the orders 6k + 1 occur for a symmetrical, three-phase winding, where $k = 0, \pm 1, \pm 2,...$ The positive and negative harmonics have different directions of motion. The number of grooves per pole and per phase and the contraction of the winding step are described by means of the so-called winding coefficients k_{win_ν}. In the majority of cases, the h.s.h. are disregarded when the electromagnetic processes in a MHD machine are investigated (Ref. 2-5). In many cases, this is valid - for example, in the case of large non-magnetic clearances, and a sufficiently thick conductor layer. Due to the smallness of the clearance between the rotor and the stator, the h.s.h. play a significant role in asynchronous motors. Therefore, special measures should be taken to suppress them. With respect to MHD machines, the influence of the h.s.h. in a molten metal and in a stator upon electrodynamic force density and energy losses has not been studied sufficiently (Ref. 6-11).

I. M. Postnikov (Ref. 12) has presented a method for computing the losses from h.s.h. in the rotor of an asynchronous machine. His method may be employed to calculate the losses in a MHD machine with a one-sided inductor for a sufficiently thick layer of molten metal (theoretically infinite) (Figure 1). However, this method is not applicable for calculating losses in a MHD /45 machine with a two-sided inductor. In the case of the two-sided inductor, the layer of molten metal has a finite thickness in contrast to the infinitely thick layer (Ref. 12). The windings of the stator with magnetic circuits are located on both sides of the molten metal layer.

The electromagnetic fields in the molten metal of a MHD machine may differ greatly in the case of a two-sided and a one-sided inductor of the magnetic field (Ref. 6). In addition, the determination of the expansion coefficients (Ref. 12) of linear current loading in series with respect to individual harmonics, when there are grooves on the surface of the stator, is an independent

problem (Ref. 18). These coefficients were assumed to be known on the basis of the theory of electric machines in (Ref. 12).

The influence of h.s.h. on the density of the electrodynamic forces and Joule heat losses in the model of a MHD machine with a two-sided symmetrical inductor was investigated in (Ref. 10). It was shown in this report that in several cases the h.s.h. makes a significant contribution to the energy losses and to the density of the electrodynamic forces.

The influence of a non-uniform air gap upon the motion of the rotor of an asynchronous machine was also investigated in (Ref. 1). This problem may be reduced to determining the induction of the magnetic field on the surface of a smooth medium (rotor) with a magnetic permeability of $\mu = \infty$ and electric conductivity of $\sigma = 0$, when there is one infinitely deep groove (Ref. 13) or an infinite series of grooves (Ref. 14) in another medium (in the steel of the stator) also with $\mu = \infty$ and $\sigma = 0$ (Figures 2 and 3).

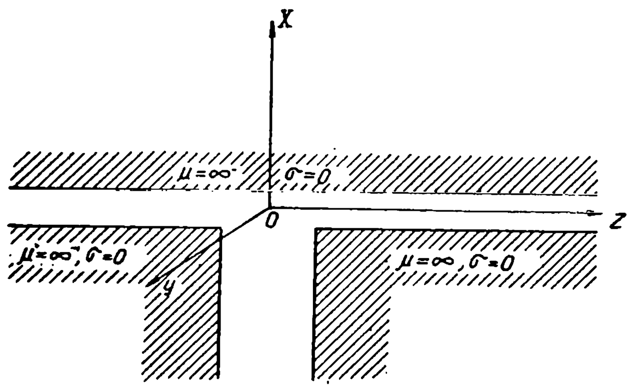

Figure 2

Infinite Groove in a Magnetic Circuit Opposite a Smooth Magnetic Circuit (Ref. 1)

For the theoretical computations, it is assumed that the dimensions of the portions of the systems shown in Figures 2 and 3 along the z-and y-axes are infinite. /46

The difference of the magnetic potentials between a smooth and serrated medium with $\mu = \infty$ and $\sigma = 0$ is assumed to be constant. The problem may be solved by the method of conformal mapping on the basis of the theory of the scalar magnetic potential. The presence of grooves leads to a decrease in the average magnitude of the induction on the smooth surface of the "rotor". This is taken into account by the Carter coefficient in the first approximation in practical applications (Ref. 1). The exact distribution of the field in the clearance is determined in (Ref. 14). However, in view of the cumbersome nature of the final results, the amplitude of separate harmonics corresponding to the projections was calculated only in this special case by Freeman (Ref. 16).

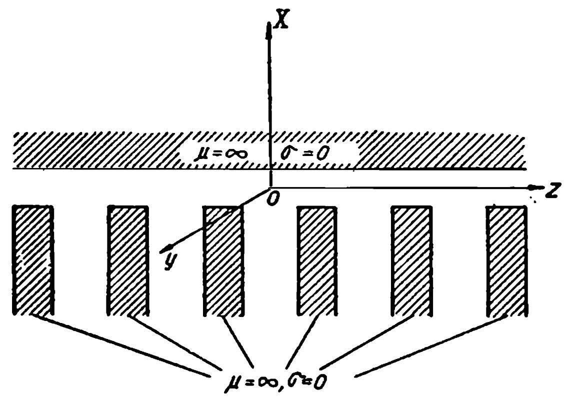

Figure 3

Infinite System of Grooves and Projections Opposite a Smooth Magnetic Circuit (Ref. 14)

A.I. Vol'dek (Ref. 15) has provided a simplified method for calculating these harmonics. For example, S. P. Pecheritsa (Ref. 17) has employed his methods. However, even for asynchronous motors it is impossible to assume that the problem of serrated pulsations has been studied exhaustively. The problem remains open regarding the distribution of the field within the clearance, and not only on the surface of the "rotor", regarding the influence of the depth and width of the grooves upon this distribution, as well as other problems. A conductive medium is located within the clearance in a MHD machine. The distribution of the field in this medium, taking the non-uniformity of the clearance into account, is also of interest. This distribution influences the density of electrodynamic forces and energy losses in an induction MHD machine. A theoretical investigation of this problem was performed in (Ref. 18), and some of the results derived in this study will be investigated below.

2. H.s.h. From the Spatial Distribution of the Winding with a Smooth Steel Surface of the Stator /47

There are several factors which complicate a theoretical investigation of the influence of h.s.h. upon the electromagnetic processes in an induction MHD machine. Any real MHD machine has finite dimensions, which completely determine the properties of the materials from which the channel of the MHD machine, the stator, and the winding are made. The velocity of the molten metal in the channel of the MHD machine is not constant over the cross-section. It is impossible to make strict allowance for all of these factors. Therefore, several simplifications must be employed in theoretical computations. The finite dimensions of the MHD machine are disregarded in an investigation of the role of h.s.h.

Let us investigate the simplified model shown in Figure 4.

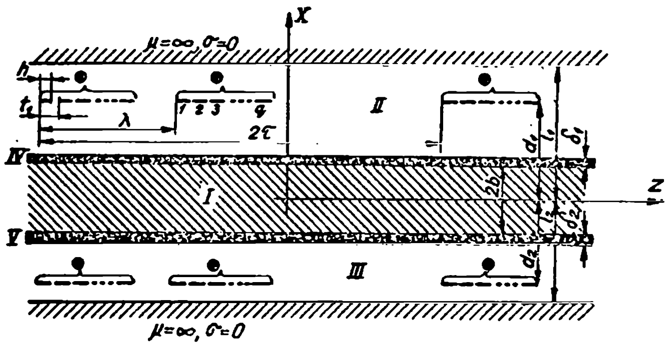

Figure 4

Non-Symmetrical Model of a MHD Transformer with Conductive Channel Walls

Here, region I is the working component of the MHD machine with conductivity σ and magnetic permeability μ_0. The regions II and III represent the non-magnetic clearance.

The winding of the stator is usually located in special grooves of the stator steel. The grooves make the non-magnetic clearance of the MHD machine non-uniform, but we shall first investigate the smooth steel surface of the stator (the influence of the grooves will be investigated in section three). In this connection, let us place the winding of the model of the stator (see Figure 4) in the clearance between the channel of the MHD machine and the stator. By changing the distances d_1 and d_2, we may obtain any arrangement of the winding in the clearance. Thus, in the case of $d_1 = \ell_1$ and $d_2 = \ell_2$ the winding lies on the steel surface of the stator. The winding (only one phase is shown) consists of q linear plates having the thickness h (q grooves per pole and per phase). The distance between the plates in one phase is $t_1 - h$ (the width of the "projection" is $t_1 - h$). The distance between the for- /48
ward and inverse currents of one phase is λ; the polar step is τ; (the ratio $\frac{\lambda}{\tau}$ characterizes the contraction of the winding step).

Regions IV and V in Figure 4 are the channel walls of an induction MHD machine with conductivity σ_c, magnetic permeability μ_0, and thickness δ_1 and δ_2. It is assumed that velocity of the medium of region I is identical at every point and equals $\mathbf{v}(0,0,v)$. It is assumed that the current is a sinusoidal current with the angular frequency ω, and is uniformly distributed over all plates of one phase. The current amplitude of one phase of the winding in region II is I_{01}; in region III it is I_{02}. The phase shift between

the currents in the upper and lower inductors is ϕ. A medium with $\mu = \infty$ and $\sigma = 0$ is located behine the winding. The other geometric dimensions are shown in the figure.

After a theoretical investigation of the model of the MHD machine shown in Figure 4 (see appendix 1), we may find the z-component (which is averaged over time and the coordinates) of the force density

$$f_z = \sum_{\nu} f_{z\nu} \tag{1}$$

the x-component (which is averaged over time and z-coordinate) of the force density

$$f_x = \sum_{\nu} f_{x\nu}(x), \tag{2}$$

where $\nu = 6k + 1$, $k = 0, \pm 1, \pm 2, \ldots$; f_ν gives the force density of the ν^{th} harmonics of the traveling magnetic field. $f_{z\nu}$ may be positive or negative, depending on the slipping. The mean Joule heat losses Q_{Joule} in the channel of the MHD machine may be obtained by means of the x-component of the Poynting vector:

$$\operatorname{Re} S_x = \frac{1}{\sigma\mu_0} f_x. \tag{3}$$

The energy losses in the molten metal* are

$$Q_{Jo\,I} = \operatorname{Re}[S_{xI}|_{x=b} - S_{xI}|_{x=-b}], \tag{4}$$

in the channel walls

$$Q_{Jo}^{IV} = \operatorname{Re}[S_{xIV}|_{x=b+\delta_1} - S_{xIV}|_{x=b}], \tag{5}$$

$$Q_{Jo}^{V} = \operatorname{Re}[S_{xV}|_{x=-b-\delta_1} - S_{xV}|_{x=-b}], \tag{6}$$

where Re designates the real part.

In numerical calculations of the force density and Joule heat losses in an induction MHD machine, it is advantageous to introduce the relative force density and Joule losses: /49

$$\bar{f}_z = f_z \frac{2b}{\mu_0 I_0^2}, \tag{7}$$

$$\bar{Q} = Q_{Jo} \frac{\sigma b}{2I_0^2}. \tag{8}$$

The contribution made by the h.s.h. to the force density and Joule heat losses is the difference between the sum of the series and the first term of

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* Energy losses in a vertical column with unit transverse cross-section.

the series (1), (2).

The computations of the force density and Joule heat losses according to formulas (7) and (8) are cumbersome. The computations were performed on a BESM-2 in the Computing Center of Lenningrad State University imenii P. Stuchok for the special case when

$$\delta_1=\delta_2=0,\quad d_1=d_2=l_1=l_2=d,\quad \lambda=\tau,\quad h=0,$$

$$\varphi=0,\quad I_{01}=I_{02}=I_0,\quad t_1=\frac{\tau}{3q},\quad q=1;\ 3;\ 5,$$

and it was found that the contribution made by the h.s.h. to the force density and Joule heat losses may be quite significant for specific values of the parameters. In this special case, $\bar{f}_z$ and $\bar{Q}_{Jo}$ were expressed by means of the following dimensionless parameters:

$$\zeta=\frac{d-b}{\tau},\quad \varepsilon=\frac{\mu_0\sigma\omega\tau^2}{\pi^2},\quad \xi=\frac{\tau}{b},$$

$$s=1-\frac{v\pi}{\omega\tau}\ \text{and}\ q. \tag{9}$$

The computations of $\bar{f}_z$ and $\bar{Q}_{Jo}$ were performed for a constant linear current density $J_0 = \frac{3I_0}{\tau}$. We shall present some results derived from the numerical calculation. Thus, Table 1 presents the relative force density in the case of $\varepsilon = 1$, $\xi = 20$, $q = 1$ and different values of ζ and s. Table 2 presents the relative Joule heat losses in the channel of a MHD machine for the same values of the parameters. As may be seen, the contribution made by the h.s.h. to the force density and the Joule heat losses depends on the paramteter ζ. The role of the h.s.h. is greatest for small values of the parameter ζ. Thus, for example, in the case of $\zeta = 10^{-3}$, $\xi = 20$, $\varepsilon = 1$, $q = 1$, $s = 1$, $\bar{f}_{z1}$ comprises only 72% of $\bar{f}_z$. The situation is the same for $\bar{Q}_{Jo}$ and $\bar{Q}_{Jo\ 1}$. In the case of $\xi > 0.1$, it is practically impossible to take into account the influence of the h.s.h.. It must be noted that in the case of s = 1 - i.e., when the molten metal is immobile - the contribution made by the h.s.h. is minimal.

It thus follows that it is impossible to estimate the influence of the h.s.h. in the case of s = 1; it will be too low. The tables present $\bar{f}_z$ and $\bar{Q}_{Jo}$ in the case of q = 1, which corresponds to the case in which the contribution made by the h.s.h. is greatest. For small ζ, q > 1 in real MHD machines. An increase in q leads to a reduction in the role of the h.s.h.. This may be readily seen from the graphs shown in Figures 5 and 6. /51

Thus, for example, Figure 5 presents a graph showing the dependence of the dimensionless force density $\bar{f}_z$ on the slipping (sum of the series and first term of the series) for the following values of the dimensionless parameters

TABLE 1

ζ	10^{-3}		$5\cdot10^{-3}$		10^{-2}		$5\cdot10^{-2}$		10^{-1}	
s	$\bar{f}_z$	$\bar{f}_{z1}$	$\bar{f}_z$	$\bar{f}_{z1}$	$\bar{f}_z$	$\bar{f}_{z1}$	$\bar{f}_z$	$\bar{f}_{z1}$	$\bar{f}_z$	$\bar{f}_{z1}$
2	4,609	3,867	4,112	3,668	3,732	3,426	1,995	1,950	1,032	1,025
1	3,936	3,970	3,555	3,583	3,140	3,163	1,347	1,352	0,616	0,617
0,5	2,273	2,694	2,087	2,352	1,818	2,006	0,718	0,749	0,320	0,325
0	−0,805	0,000	−0,497	0,000	−0,348	0,000	−0,056	0,000	−0,009	0,000
−0,5	−3,870	−2,694	−3,073	−2,352	−2,509	−2,006	−0,829	−0,749	−0,339	−0,325
−1	−5,503	−3,970	−4,516	−3,583	−3,814	−3,163	−1,456	−1,352	−0,635	−0,617
−2	−6,064	−3,867	−4,987	−3,668	−4,342	−3,426	−2,095	−1,950	−1,049	−1,025

TABLE 2

ζ	10^{-3}		$5\cdot10^{-3}$		10^{-2}		$5\cdot10^{-2}$		10^{-1}	
s	$\bar{Q}_{Jo}$	$\bar{Q}_{Jo}$	$\bar{Q}_{Jo}$	$\bar{Q}_{Jo}$	$\bar{Q}_{Jo}$	$\bar{Q}_{Jo}$	$\bar{Q}_{Jo}$	$\bar{Q}_{Jo}$	$\bar{Q}_{Jo}$	$\bar{Q}_{Jo}$
2	−0,332	−0,304	−0,305	−0,288	−0,281	−0,269	−0,155	−0,153	−0,081	−0,080
1	−0,156	−0,156	−0,141	−0,141	−0,125	−0,124	−0,053	−0,053	−0,024	−0,024
0,5	−0,062	−0,053	−0,052	−0,046	−0,044	−0,039	−0,015	−0,015	−0,006	−0,006
0	−0,033	0,000	−0,021	0,000	−0,015	0,000	−0,002	0,000	0,000	0,000
−0,5	−0,124	−0,053	−0,091	−0,046	−0,071	−0,039	−0,020	−0,015	−0,007	−0,006
−1	−0,279	−0,156	−0,216	−0,141	−0,177	−0,124	−0,062	−0,053	−0,026	−0,024
−2	−0,566	−0,304	−0,446	−0,288	−0,380	−0,269	−0,171	−0,153	−0,083	−0,080

(Ref. 10): $\zeta = 10^{-2}$, $\varepsilon = 3.72$, $\xi = 10$, $q = 1; 3;5$.

Figure 6 shows the dependence of the Joule heat losses on the slipping for the same values of the dimensionless parameters. It should again be noted that the first term of the series of the force density and the Joule heat losses differs the least from the sum of the entire series in the case of s = 1. Thus, for example, in the case of s = 1 the entire force density practically coincides with the first term of the series (1), whereas in the case of s = −2

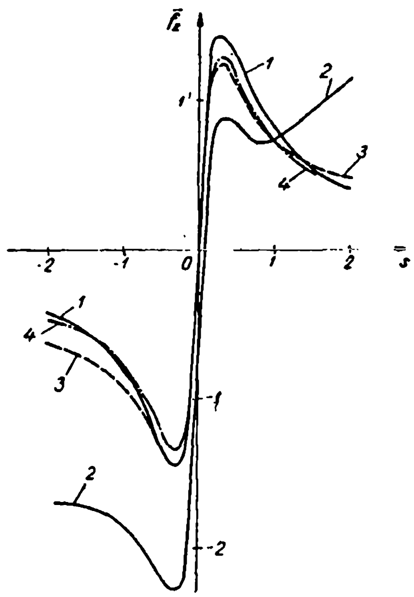

Figure 5

1 - First Term of the Series $\bar{f}_{z1}$ in the Case of q = 1;
2 - $\bar{f}_{z1}$ in the Case of q = 1;
3 - $\bar{f}_z$ in the Case of q = 3;
4 - $\bar{f}_z$ in the Case of q = 5.

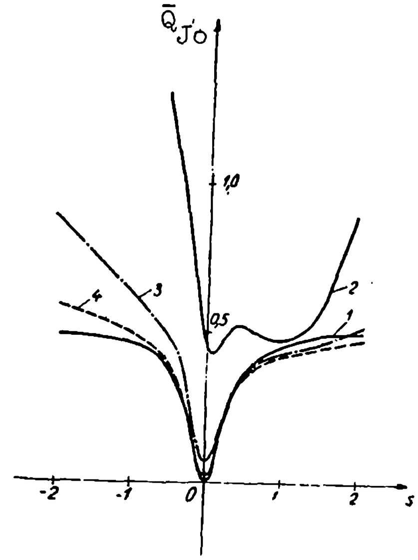

Figure 6

1 - First Term of the Series $\bar{Q}_{Jo\ 1}$ in the Case of q = 1; 2 - $\bar{Q}_{Jo}$ in the Case of q = 1; 3 - $\bar{Q}_{Jo}$ in the Case of q = 3; 4 - $\bar{Q}_{Jo}$ in the Case of q = 5.

(generator regime) and in the case of q = 1 the difference is a factor of four. In the case of q = 3 the difference is a factor of 1.62, and in the case of q = 5 -- 1.24 (see Figure 5). The situation is the same for the Joule heat losses (see Figure 6).

It follows from the tables and graphs presented above that allowance /52 for only the first harmonics of a traveling magnetic field does not always lead to a correct result. A rigorous distinction must be drawn between the region of the parameters (ζ, s, q, etc.), where it is possible to disregard the influence of the h.s.h. and where it is impossible to do so.

3. Influence of an Inductor Surface Having Projections on the Electromagnetic Field Distribution in a Conductive Band

The grooves and projections on the steel surface of the stator deform the electromagnetic field in the molten metal of a MHD machine. This distorted

field may be expanded in series and we thus obtain the so-called harmonics "corresponding to the projections" (Ref. 15). However, this expansion is rendered more difficult by the fact that the "distorted field" is unknown, which must be expanded in series. In the air gap of asynchronous machines, this field may be found under the following assumptions (Ref. 14):

(1) The rotor and the stator have the conductivity $\sigma = 0$ and magnetic permeability $\mu = \infty$ (see Figure 3);

(2) The grooves on the surface of the stator are infinite or finite in depth;

(3) The rotor may be smooth or may have grooves;

(4) A constant difference of the magnetic potentials is given between the rotor and the stator;

(5) The dimensions of the system (Figure 3) along the y- and z-axes are infinite.

The definitive results derived from the study by Coe and Taylor (Ref. 14) are not suitable for numerical calculations. In practice, the method of Vol'dek (Ref. 15) is employed more frequently to determine the amplitudes of the harmonics corresponding to the projections. In this method, the grooves on the rotor surface influence the magnetic permeability of the air gap. The expansion in series of the approximate periodic curve of the magnetic conductivity leads to the harmonics corresponding to the projections. If allowance is only made for the first term of the expansion, this produces a decrease in the average induction in the gap due to the non-uniformity of the air gap by a factor of k_δ -- where k_δ is the Carter coefficient (Ref. 1). For the most part, in practice the non-uniformity of the air gap is taken into account by means of this coefficient -- i.e., it is assumed that it is equivalent to a certain increase in the uniform gap.

There is a conductive medium in the air gap in MHD machines. The electromagnetic field in this medium differes greatly from the electromagnetic field in the air gap of an asynchronous machine. The depth of the grooves on the steel surface of the stator has a finite value. It is not sufficient to take into account the non-uniformity of the air gap in the MHD machine by means of the Carter coefficient.

In this connection, the study (Ref. 18) discussed an approximate model of the MHD machine (Figure 7) in order to allow for the influence of the non-uniformity of the air gap upon electromagnetic processes in the MHD machine. This model has infinite dimensions along the y- and z-axes. Regions III and V have the conductivity $\sigma = 0$, and the magnetic permeability $\mu = \mu_0$. Region IV is the conductor with the conductivity σ and magnetic permeability μ_0. Region I is a groove occupied by an alternating current with constant density amplitude j_0 and with angular frequency ω; regions II and II' are the /53

empty grooves. The vertical lines $z = -\frac{\tau}{2}$ and $z = \frac{\tau}{2}$ divide half of the period (the system has a period along the z-axis equalling 2τ). The geometric dimensions of the regions are shown in the figure.

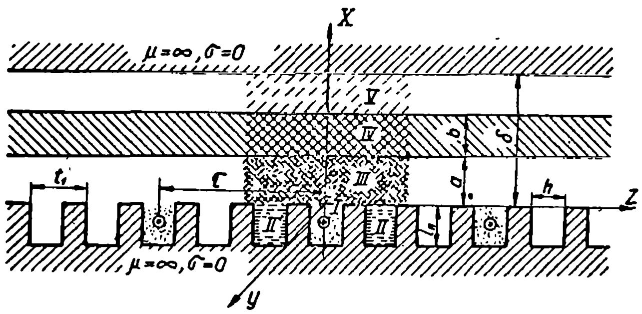

Figure 7

Model of the MHD Transformer with a One-Sided Inductor, with Allowance for Non-uniform Gap

The empty grooves are left for loops of the winding for the second and third phases. This system corresponds to a one-sided inductor having a tri-phase current with the number of grooves per pole and phase equalling unity. The solutions obtained make it possible to write the expression of the vector potential for a two-sided symmetrical inductor (Ref. 18) and in other special cases.

According to (Ref. 18), the vector potential A(0,A,0)* in different regions (see Figure 7) has the following form.

$$A_{\mathrm{I}} = \sum_{m=0}^{\infty}\sum_{k=0}^{\infty}(2K_k\varphi_k\alpha_{mk} + \mu_0 j_0 l_n{}^2\delta_{m0})\frac{\operatorname{ch}\lambda_m(x+l_n)}{\operatorname{ch}\lambda_m l_n}\cos\lambda_m z - -\frac{1}{2}\mu_0 j_0(x+l_n)^2, \quad (10)$$

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$$A_{\mathrm{II}} = \sum_{m=0}^{\infty}\sum_{k=0}^{\infty}K_k\varphi_k\beta_{mk}\frac{\operatorname{ch}\frac{1}{2}\lambda_m(x+l_n)}{\operatorname{ch}\frac{1}{2}\lambda_m l_n}\cos\pi m\left(\frac{\tau-3z}{3h}-\frac{1}{2}\right), \quad (11)$$

$$A_{\mathrm{II'}} = \sum_{m=0}^{\infty}\sum_{k=0}^{\infty}K_k\varphi_k\beta_{mk}\frac{\operatorname{ch}\frac{1}{2}\lambda_m(x+l_n)}{\operatorname{ch}\frac{1}{2}\lambda_m l_n}\cos\pi m\left(\frac{\tau+3z}{3h}-\frac{1}{2}\right), \quad (12)$$

* The dependence on time $e^{i\omega t}$ is excluded in all formulas for the vector potential.

$$A_{III} = \sum_{m=0}^{\infty} K_m \Big\{ \operatorname{ch} \alpha_m (x-\delta+b) \operatorname{ch} \gamma_m b + $$

$$+ \Big[\frac{\alpha_m}{\gamma_m} \operatorname{sh} \alpha_m (\delta - a - b) \operatorname{ch} \alpha_m (x-a) - $$

$$- \frac{\gamma_m}{\alpha_m} \operatorname{ch} \alpha_m (a+b-\delta) \operatorname{sh} \alpha_m (x-a) \Big] \operatorname{sh} \gamma_m b \Big\} \frac{\cos \alpha_m z}{\operatorname{ch} \alpha_m \delta}, \tag{13}$$

$$A_{IV} = \sum_{m=0}^{\infty} K_m \Big[\operatorname{ch} \alpha_m (a+b-\delta) \operatorname{ch} \gamma_m (a+b-x) - \tag{14}$$

$$- \frac{\alpha_m}{\gamma_m} \operatorname{sh} \alpha_m (\delta - a - b) \operatorname{sh} \gamma_m (x-a-b) \Big] \frac{\cos \alpha_m z}{\operatorname{ch} \alpha_m \delta}, \tag{15}$$

$$A_V = \sum_{m=0}^{\infty} K_m \frac{\operatorname{ch} \alpha_m (x-\delta)}{\operatorname{ch} \alpha_m \delta} \cos \alpha_m z,$$

where

$$\lambda_m = \frac{2\pi}{h} m; \quad \alpha_m = \frac{\pi}{\tau} (2m+1); \tag{16}$$

$$\gamma_m = \sqrt{\alpha_m^2 + i \sigma \mu_0 \omega}; \tag{17}$$

$$\varphi_m = \Big\{ \operatorname{ch} \alpha_m (b-\delta) \operatorname{ch} \gamma_m b + \Big[\frac{\alpha_m}{\gamma_m} \operatorname{sh} \alpha_m (\delta - a - b) \operatorname{ch} \alpha_m a + \tag{18}$$

$$+ \frac{\gamma_m}{\alpha_m} \operatorname{ch} \alpha_m (\delta - a - b) \operatorname{sh} \alpha_m a \Big] \operatorname{sh} \gamma_m b \Big\} \frac{1}{\operatorname{ch} \alpha_m \delta};$$

$$\psi_m = \Big\{ \operatorname{sh} \alpha_m (b-\delta) \operatorname{ch} \gamma_m b - \Big[\frac{\alpha_m}{\gamma_m} \operatorname{sh} \alpha_m (\delta - a - b) \operatorname{sh} \alpha_m a + $$

$$+ \frac{\gamma_m}{\alpha_m} \operatorname{ch} \alpha_m (\delta - a - b) \operatorname{ch} \alpha_m a \Big] \operatorname{sh} \gamma_m b \Big\} \frac{1}{\operatorname{ch} \alpha_m \delta}; \tag{19}$$

$$\alpha_{mk} = \frac{1}{\pi} \cdot \frac{\varepsilon_k}{\varepsilon_k^2 - m^2} \cos \pi m \sin \pi \varepsilon_k; \tag{20}$$ /55

$$\beta_{mk} = \frac{1}{\pi} \cdot \frac{4\varepsilon_k}{4\varepsilon_k^2 - m^2} \Big[\cos \pi m \sin \pi \Big(\varepsilon_k + \frac{2k+1}{3} \Big) + \tag{21}$$

$$+ \sin \pi \Big(\varepsilon_k - \frac{2k+1}{3} \Big) \Big];$$

$$\varepsilon_k = \frac{h}{2\tau} (2k+1). \tag{22}$$

The coefficients K_m are determined from an infinite system of equations

$$\sum_{m=0}^{\infty} K_m \Theta_{km} = 2\mu_0 j_0 l_n \varkappa_k, \tag{23}$$

where

$$\Theta_{km} = \varphi_m \sum_{\nu=1}^{\infty} \lambda_\nu \left(4\alpha_{\nu m}\alpha_{\nu k} \operatorname{th} \lambda_\nu l_n + \beta_{\nu m}\beta_{\nu k} \operatorname{th} \frac{1}{2}\lambda_\nu l_n\right) - \tag{24}$$

$$- \frac{\tau}{h} \alpha_m \psi_m \delta_{km};$$

$$\varkappa_k = \frac{\sin \pi\varepsilon_k}{\pi\varepsilon_k}; \tag{25}$$

$$\delta_{km} = \begin{cases} 0; & k \neq m, \\ 1; & k = m. \end{cases} \tag{26}$$

Thus, the determination of the vector potential may be reduced to solving an infinite system of equations (23). Let us investigate certain special cases (Ref. 18).

1. There is no conductive band IV in the clearance. In order to determine the vector potential in this case, we may set $\sigma = 0$. As a result, we have

$$\varphi_m = 1, \quad \psi_m = -\operatorname{th} \alpha_m \delta, \quad \gamma_m = \alpha_m,$$

$$\Theta_{km} = \sum_{\nu=1}^{\infty} \lambda_\nu \left(4\alpha_{\nu m}\alpha_{\nu k} \operatorname{th} \lambda_\nu l_n + \beta_{\nu m}\beta_{\nu k} \operatorname{th} \frac{1}{2}\lambda_\nu l_n\right) +$$

$$+ \frac{\tau}{h} \alpha_m \operatorname{th} \alpha_m \delta \delta_{km},$$

$$A_{III} = A_{IV} = A_V = \sum_{m=0}^{\infty} K_m \frac{\operatorname{ch} \alpha_m (x-\delta)}{\operatorname{ch} \alpha_m \delta} \cos \alpha_m z. \tag{27}$$

II. In the case of the two-sided symmetrical inductor, the expression for the vector potential may be written as follows: /56

$$A_{III} = \sum_{m=0}^{\infty} K_m \left[\operatorname{ch} \alpha_m (x-a) \operatorname{ch} \gamma_m b - \right. \tag{28}$$

$$\left. - \frac{\gamma_m}{\alpha_m} \operatorname{sh} \alpha_m (x-a) \operatorname{sh} \gamma_m b \right] \frac{\cos \alpha_m z}{\operatorname{ch} \alpha_m \delta},$$

$$A_{IV} = \sum_{m=0}^{\infty} K_m \operatorname{ch} \gamma_m (\delta - x) \frac{\cos \alpha_m z}{\operatorname{ch} \alpha_m \delta}. \tag{29}$$

The coefficients K_m are again determined from system (23), where

$$\varphi_m = \left[\operatorname{ch} \alpha_m a \operatorname{ch} \gamma_m (\delta - a) + \frac{\gamma_m}{\alpha_m} \operatorname{sh} \alpha_m a \operatorname{sh} \gamma_m (\delta - a)\right] \frac{1}{\operatorname{ch} \alpha_m \delta}, \tag{30}$$

$$\psi_m=\left[-\operatorname{sh}\alpha_m a\operatorname{ch}\gamma_m(\delta-a)+\frac{\gamma_m}{\alpha_m}\operatorname{ch}\alpha_m a\operatorname{sh}\gamma_m(\delta-a)\right]\frac{1}{\operatorname{ch}\alpha_m\delta}. \tag{31}$$

Formulas (28) -- (31) were obtained under the assumption that a + b = = δ -- i.e., in the case of a two-sided symmetrical inductor, the horizontal plane of symmetry passing through the middle of the conductive band corresponds to the boundary of the medium with $\mu = \infty$ and $\sigma = 0$ in the case of $x = \delta$. The induction vector of the magnetic field $\mathbf{B}$ = rot $\mathbf{A}$ is perpendicular to this plane of symmetry.

If there is no medium with $\mu = \infty$ and $\sigma = 0$ above the conductive band, we have the following expression for the vector potential (Ref. 18):

$$A_{III}=\sum_{m=0}^{\infty}K_m e^{-\alpha_m(a+b)}\left\{e^{-\alpha_m(x-a)}\operatorname{ch}\gamma_m b+\right.$$
$$\left.+\left[\frac{\alpha_m}{\gamma_m}\operatorname{ch}\alpha_m(x-a)-\frac{\gamma_m}{\alpha_m}\operatorname{sh}\alpha_m(x-a)\right]\operatorname{sh}\gamma_m b\right\}\cos\alpha_m z, \tag{32}$$

$$A_{IV}=\sum_{m=0}^{\infty}K_m e^{-\alpha_m(a+b)}\left[\operatorname{ch}\gamma_m(a+b-x)-\right.$$
$$\left.-\frac{\alpha_m}{\gamma_m}\operatorname{sh}\gamma_m(x-a-b)\right]\cos\alpha_m z, \tag{33}$$

$$A_V=\sum_{m=0}^{\infty}K_m e^{-\alpha_m x}\cos\alpha_m z. \tag{34}$$

The coefficients K_m, as always, are determined from the system /57
(23), where

$$\varphi_m=e^{-\alpha_m(a+b)}\left\{e^{\alpha_m a}\operatorname{ch}\gamma_m b+\left[\frac{\alpha_m}{\gamma_m}\operatorname{ch}\alpha_m a+\frac{\gamma_m}{\alpha_m}\operatorname{sh}\alpha_m a\right]\operatorname{sh}\gamma_m b\right\}, \tag{35}$$

$$\psi_m=-e^{-\alpha_m(a+b)}\left\{a^{\alpha_m a}\operatorname{ch}\gamma_m b+\left[\frac{\alpha_m}{\gamma_m}\operatorname{sh}\alpha_m a+\frac{\gamma_m}{\alpha_m}\operatorname{ch}\alpha_m a\right]\operatorname{sh}\gamma_m b\right\}. \tag{36}$$

Thus, it is possible to find the vector potential in other special cases. It must be pointed out that when the depth of the grooves ℓ_n strives to zero -- so that the total current in the groove does not change -- i.e.,

$$j_0 l_n=\frac{I_0}{h}=\text{const},$$

the expression for the vector potential changes into a similar one for an inductor without grooves and projections (Ref. 6-10).

Summing up the vector potential of three phases, we may obtain the traveling magnetic field, with allowance for the non-uniformity of the air

gap.

In the final analysis, determination of the influence of grooves and projections may be reduced to an investigation of an infinite system of equations (23). This system requires additional analysis, in order to establish the rapid decrease in the coefficients K_m when m increases, and the analysis requires additional computational work. It may be performed most readily on computers.

APPENDIX

4. Results of Analytical Computations

The results derived in (Ref. 11) are employed to determine the distribution of the force density and the Joule heat losses in the channel and in the walls of the model of the induction MHD machine (see Figure 4). A solution of the equation for the vector potential A(0,A,0) was obtained in this study:

$$A_1 = 3\sum_{\nu}\left[a_\nu^{(1)} e^{-\gamma_\nu x} + a_\nu^{(2)} e^{\gamma_\nu x}\right] e^{i(\omega s_\nu t - \alpha_\nu z')}*,$$

$$A_{1\mathrm{V}} = 3\sum_{\nu}\left[g_\nu^{(1)} e^{-\gamma_\nu^c x} + g_\nu^{(2)} e^{\gamma_\nu^c x}\right] e^{i(\omega t - \alpha_\nu z)},$$

$$A_{\mathrm{V}} = 3\sum_{\nu}\left[c_\nu^{(1)} e^{-\gamma_\nu^c x} + c_\nu^{(2)} e^{\gamma_\nu^c x}\right] e^{i(\omega t - \alpha_\nu z)},$$

where /58

$$z' = z - vt; \quad s_\nu = 1 - \frac{v\pi}{\omega\tau}\nu;$$

$$\alpha = \frac{\pi}{\tau}; \quad \alpha_\nu = \alpha\nu; \quad \gamma_\nu = \sqrt{\alpha_\nu^2 + i\sigma\mu_0\omega s_\nu};$$

$$\gamma_\nu^c = \sqrt{\alpha_\nu^2 + i\sigma_c\mu_0\omega}; \quad \nu = 6k+1; \quad k = 0, \pm 1, \pm 2, \ldots$$

$$a_\nu^{(1)} = K_{wi\nu} \frac{\mu_0 I_{02} e^{i\varphi} \operatorname{ch} \alpha_\nu (l_2 - d_2) N_\nu + \mu_0 I_{01} \operatorname{ch} \alpha_\nu (l_1 - d_1) L_\nu}{\tau (G_\nu L_\nu - M_\nu N_\nu)};$$

$$a_\nu^{(2)} = K_{wi\nu} \frac{\mu_0 I_{01} \operatorname{ch} \alpha_\nu (l_1 - d_1) M_\nu + \mu_0 I_{02} e^{i\varphi} \operatorname{ch} \alpha_\nu (l_2 - d_2) G_\nu}{\tau (M_\nu N_\nu - G_\nu L_\nu)};$$

$$g_\nu^{(1)} = \frac{1}{2}\left[\left(1 + \frac{\gamma_\nu}{\gamma_\nu^c}\right) a_\nu^{(1)} e^{(\gamma_\nu^c - \gamma_\nu)b} + \left(1 - \frac{\gamma_\nu}{\gamma_\nu^c}\right) a_\nu^{(2)} e^{(\gamma_\nu^c + \gamma_\nu)b}\right];$$

$$g_\nu^{(2)} = \frac{1}{2}\left[\left(1 - \frac{\gamma_\nu}{\gamma_\nu^c}\right) a_\nu^{(1)} e^{-(\gamma_\nu^c + \gamma_\nu)b} + \left(1 + \frac{\gamma_\nu}{\gamma_\nu^c}\right) a_\nu^{(2)} e^{(\gamma_\nu - \gamma_\nu^c)b}\right];$$

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* The z' axis is connected with the moving medium I.

$$c_\nu^{(1)}=\frac{1}{2}\left[\left(1+\frac{\gamma_\nu}{\gamma_\nu^c}\right)a_\nu^{(1)}e^{(\gamma_\nu-\gamma_\nu^c)b}+\left(1-\frac{\gamma_\nu}{\gamma_\nu^c}\right)a_\nu^{(2)}e^{-(\gamma_\nu+\gamma_\nu^c)b}\right];$$

$$c_\nu^{(2)}=\frac{1}{2}\left[\left(1-\frac{\gamma_\nu}{\gamma_\nu^c}\right)a_\nu^{(1)}e^{(\gamma_\nu+\gamma_\nu^c)b}+\left(1+\frac{\gamma_\nu}{\gamma_\nu^c}\right)a_\nu^{(2)}e^{(\gamma_\nu^c-\gamma_\nu)b}\right];$$

$$M_\nu=\frac{\gamma_\nu^c+\gamma_\nu}{2\gamma_\nu^c}e^{\gamma_\nu b+\gamma_\nu^c\delta_2}[\alpha_\nu \operatorname{sh}\alpha_\nu(b+\delta_2-l_2)-$$
$$-\gamma_\nu^c \operatorname{ch}\alpha_\nu(b+\delta_2-l_2)]+\frac{\gamma_\nu^c-\gamma_\nu}{2\gamma_\nu^c}e^{\gamma_\nu b-\gamma_\nu^c\delta_2}[\alpha_\nu \operatorname{sh}\alpha_\nu(b+\delta_2-l_2)+$$
$$+\gamma_\nu^c \operatorname{ch}\alpha_\nu(b+\delta_2-l_2)];$$

$$G_\nu=\frac{\gamma_\nu^c+\gamma_\nu}{2\gamma_\nu^c}e^{-(\gamma_\nu b+\gamma_\nu^c\delta_1)}[\alpha_\nu \operatorname{sh}\alpha_\nu(l_1-b-\delta_1)-$$
$$-\gamma_\nu^c \operatorname{ch}\alpha_\nu(l_1-b-\delta_1)]+\frac{\gamma_\nu^c-\gamma_\nu}{2\gamma_\nu^c}e^{\gamma_\nu^c\delta_1-\gamma_\nu b}[\alpha_\nu \operatorname{sh}\alpha_\nu(l_1-b-\delta_1)+$$
$$+\gamma_\nu^c \operatorname{ch}\alpha_\nu(l_1-b-\delta_1)];$$

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$$N_\nu=\frac{\gamma_\nu^c-\gamma_\nu}{2\gamma_\nu^c}e^{\gamma_\nu b-\gamma_\nu^c\delta_1}[\alpha_\nu \operatorname{sh}\alpha_\nu(l_1-b-\delta_1)-\gamma_\nu^c \operatorname{ch}\alpha_\nu(l_1-b-\delta_1)]+$$
$$+\frac{\gamma_\nu^c+\gamma_\nu}{2\gamma_\nu^c}e^{\gamma_\nu^c\delta_1+\gamma_\nu b}[\alpha_\nu \operatorname{sh}\alpha_\nu(l_1-b-\delta_1)+\gamma_\nu^c \operatorname{ch}\alpha_\nu(l_1-b-\delta_1)];$$

$$L_\nu=\frac{\gamma_\nu^c-\gamma_\nu}{2\gamma_\nu^c}e^{\gamma_\nu^c\delta_2-\gamma_\nu b}[\alpha_\nu \operatorname{sh}\alpha_\nu(b+\delta_2-l_2)-\gamma_\nu^c \operatorname{ch}\alpha_\nu(b+\delta_2-l_2)]+$$
$$+\frac{\gamma_\nu^c+\gamma_\nu}{2\gamma_\nu^c}e^{-(\gamma_\nu b+\gamma_\nu^c\delta_2)}[\alpha_\nu \operatorname{sh}\alpha_\nu(b+\delta_2-l_2)+\gamma_\nu^c \operatorname{ch}\alpha_\nu(b+\delta_2-l_2)].$$

The winding coefficient $K_{wi\nu}$ of the harmonics of the order ν has the following form (Ref. 10):

$$K_{wi\nu}=K_{q\nu}K_{\lambda\nu}K_{h\nu},$$

where the coefficient of the winding distribution is

$$K_{q\nu}=\frac{\sin\frac{\pi\nu t_1}{2\tau}q}{q\sin\frac{\pi\nu t_1}{2\tau}},$$

the contraction coefficient of the winding step is

$$K_{\lambda\nu}=\sin\frac{\pi\lambda\nu}{2\tau},$$

and the coefficient depending on the width of the inductor plates (winding coefficient "of the groove opening" [Ref. 15]) is

$$K_{h_\nu} = \frac{2\tau}{\pi\nu h}\sin\frac{\pi\nu h}{2\tau}.$$

The complex expressions of the magnetic field induction **B** and the current density **j** may be readily obtained according to the following formulas

$$\mathbf{B} = \operatorname{rot}\mathbf{A},\quad \mathbf{j} = -\sigma\frac{\partial\mathbf{A}}{\partial t}.$$

According to (Ref. 11), for region I we have:

$$B_{zI} = 3\sum_\nu \gamma_\nu\left[a_\nu^{(2)}e^{\gamma_\nu x} - a_\nu^{(1)}e^{-\gamma_\nu x}\right]e^{i(\omega s_\nu t - \alpha_\nu z')},$$

$$B_{xI} = 3i\sum_\nu \alpha_\nu\left[a_\nu^{(2)}e^{-\gamma_\nu x} + a_\nu^{(2)}e^{\gamma_\nu x}\right]e^{i(\omega s_\nu t - \alpha_\nu z')},$$

$$j_{yI} = -3i\omega\sigma\sum_\nu s_\nu\left[a_\nu^{(1)}e^{-\gamma_\nu x} + a_\nu^{(2)}e^{\gamma_\nu x}\right]e^{i(\omega s_\nu t - \alpha_\nu z')}.$$

The expressions for the induction **B** and the current **j** in regions IV and V are similar to the corresponding expressions in region I. However, instead of the coefficients $a_\nu^{(1)}$ and $a_\nu^{(2)}$ we must substitute $g_\nu^{(1)}$ and $g_\nu^{(2)}$ (region IV) or c_ν (region V) instead of $z' - z$, $\sigma - \sigma_c$, $\gamma_\nu - \gamma_\nu^c$. All s_ν must be set equal to unity. /60

Employing the expressions obtained for the magnetic field induction and current density, we may find the force density **f** which is averaged over time and the coordinates (Ref. 9):

$$f_{zI} = \sum_\nu f_{zI\nu},$$

$$f_{xI} = \sum_\nu f_{xI\nu}(x)^*,$$

where

$$f_{zI\nu} = \frac{9\sigma\omega}{2}\alpha_\nu s_\nu |a_\nu^{(1)}|^2\left[\left(1 + \left|\frac{a_\nu^{(2)}}{a_\nu^{(1)}}\right|^2\right)\frac{\operatorname{sh}(2b\operatorname{Re}\gamma_\nu)}{2b\operatorname{Re}\gamma_\nu} + \right.$$

$$\left. + 2\operatorname{Re}\left(\frac{a_\nu^{(2)}}{a_\nu^{(1)}}\right)\frac{\sin(2\operatorname{Im}\gamma_\nu b)}{2b\operatorname{Im}\gamma_\nu}\right];$$

$$f_{xI\nu} = -\frac{9\sigma\omega}{2}\gamma_\nu^* s_\nu \operatorname{Re} i|a_\nu^{(1)}|^2\left[\left|\frac{a_\nu^{(2)}}{a_\nu^{(1)}}\right|^2 e^{2\operatorname{Re}\gamma_\nu x} - \right.$$

* f_{xI} is averaged over time and the z-coordinate.

$$-e^{-2\,\mathrm{Re}\,\gamma_\nu x}-2i\,\mathrm{Im}\left(\frac{a_\nu^{(2)}}{a_\nu^{(1)}}e^{2i\,\mathrm{Im}\,\gamma_\nu x}\right)\Bigg];$$

* is the sign of the complex conjugate. In regions IV and V, the expressions for the force density are similar. As indicated in the appendix, we need only substitute $a_\nu^{(1)}$ and $a_\nu^{(2)}$ by $g_\nu^{(1)}$ and $g_\nu^{(2)}$, etc.

Let us investigate several special cases (Ref. 2, 3, 7, 8, 10). The formulas for the force density and the Joule heat losses remain the same in every case, only the coefficients $a_\nu^{(1)}$ and $a_\nu^{(2)}$ change.

Thus, we must substitute $\delta_1 = \delta_2 = 0$, $\gamma_\nu^c = \alpha_\nu$ in the coefficients $a_\nu^{(1)}$ and $a_\nu^{(2)}$, in the case of a two-sided non-symmetrical inductor without conductive channel walls of the MHD machine. After this, we obtain the following expressions M_ν, G_ν, N_ν, L_ν for the coefficients $a_\nu^{(1)}$ and $a_\nu^{(2)}$: /61

$$M_\nu=e^{\gamma_\nu b}[\alpha_\nu\,\mathrm{sh}\,\alpha_\nu(b-l_2)-\gamma_\nu\,\mathrm{ch}\,\alpha_\nu(b-l_2)],$$

$$G_\nu=e^{-\gamma_\nu b}[\alpha_\nu\,\mathrm{sh}\,\alpha_\nu(l_1-b)-\gamma_\nu\,\mathrm{ch}\,\alpha_\nu(l_1-b)],$$

$$N_\nu=e^{\gamma_\nu b}[\alpha_\nu\,\mathrm{sh}\,\alpha_\nu(l_1-b)+\gamma_\nu\,\mathrm{ch}\,\alpha_\nu(l_1-b)],$$

$$L_\nu=e^{-\gamma_\nu b}[\alpha_\nu\,\mathrm{sh}\,\alpha_\nu(b-l_2)+\gamma_\nu\,\mathrm{ch}\,\alpha_\nu(b-l_2)].$$

In the case of a one-sided inductor without conductive channel walls of the MHD machine (Ref. 8), we must set $I_{01} = 0$, $\ell_1 = \infty$, $\delta_1 = \delta_2 = 0$, $\phi = 0$ in order to determine the coefficients $a_\nu^{(1)}$ and $a_\nu^{(2)}$. In this case, we have

$$a_\nu^{(1)}=K_{\mathrm{wi}\nu}\frac{\mu_0 I_{20}}{\tau}\cdot\frac{\mathrm{ch}\,\alpha_\nu(l_2-d_2)N_\nu}{G_\nu L_\nu-M_\nu N_\nu},$$

$$a_\nu^{(2)}=K_{\mathrm{wi}\nu}\frac{\mu_0 I_{20}}{\tau}\cdot\frac{\mathrm{ch}\,\alpha_\nu(l_2-d_2)G_\nu}{M_\nu N_\nu-G_\nu L_\nu},$$

where M_ν, G_ν, N_ν, L_ν are the same as in the case of a two-sided non-symmetrical inductor.

For a two-sided symmetrical inductor without conductive channel walls of the MHD machine (Ref. 9, 10), we must set

$$I_{01}=I_{02}=I_0,\quad \varphi=0,\quad l_1=l_2=l,\quad d_1=d_2=d,\quad \delta_1=\delta_2=0.$$

After this, we obtain

$$a_\nu^{(1)}=a_\nu^{(2)}=\frac{\mu_0 I_0}{2\tau}\cdot\frac{\mathrm{ch}\,\alpha_\nu(l-d)}{\alpha_\nu\,\mathrm{ch}\,\gamma_\nu b\,\mathrm{sh}\,\alpha_\nu(l-b)+\gamma_\nu\,\mathrm{sh}\,\gamma_\nu b\,\mathrm{ch}\,\alpha_\nu(l-b)}.$$

The method indicated above may be employed in other special cases to obtain the coefficients a_ν which determine the distribution of the electromagnetic field, the force density, and Joule heat losses in the channel of an induction MHD machine.

REFERENCES

1. Geller, B., Gamata, V. Dopolnitel'nyye polya, momenty i poteri moshchnosti v asinchronnykh mashinakh (Supplementary Fields, Moments, and Power Losses in Asynchronous Machines), Moscow, 1964.
2. Tyutin, I.A. Mekhanicheskiye sily v begushchem magnitnom pole. - V kn.: Voprosy energetiki (Mechanical Forces in a Traveling Magnetic Field. - In the Book: Problems of Energetics), 3. Izdatel'stvo AN Latv. SSR, 1955.
3. Liyelpeter, Ya.Ya., Petrovich, R.A. K teorii ploskikh induktsionnykh nasosov (Theory of Plane Induction Pumps). Izdatel'stvo AN Latv. SSR, Seriya Fizicheskikh i Tekhnicheskikh Nauk, 1, 1964.
4. Okhremenko, N.M. Opredeleniye optimal'nykh razmerov induktsionnykh nasosov (Determination of Optimum Dimensions of Induction Pumps). Elektrichestvo, 11, 1964.
5. Vol'dek, A.I. Toki i usiliya v sloye zhidkogo metalla ploskikh line- /62 ynykh induktsionnykh hasosov (Currents and Stresses in the Molten Metal Layer of Plane, Linear Induction Pumps). Izvestiya Vysshikh Uchebnykh Zavedeniy, Elektromekhanika, 1, 1959.
6. Valdmanis, Ya.Ya., Mikel'son, Yu.Ya. Nakhozhdeniye elektromagnitnogo polya v beskonechnoy provodyashchey polose v pole ploskikh beskonechnykh induktorov (Determination of the Electromagnetic Field in an Infinite Conductive Band in the Field of Plane, Infinite Inductors). Izvestiya AN Latv. SSR, Seriya Fizicheskikh i Tekhnicheskikh Nauk, 1, 1965.
7. Valdmanis, Ya.Ya., Kunin, P.Ye., Mikel'son, Yu.Ya., Taksar, I.M. Provodyashchiy sloy v begushchem electromagnitnom pole dvukhstoronnego induktora (Conductive Layer in a Traveling Electromagnetic Field of a Two-Sided Inductor). Magnitnaya Gidrodinamika, 2, 1965.
8. Valdamanis, Ya.Ya. Elektrodinamicheskiye sily, deystvuyushchiye na beskonechnuyu provodyashchuyu polosu v pole odnostoronnego induktora trekh-faznogo toka (Electrodynamic Forces Acting upon an Infinite Conductive Band in the Field of a One-Sided Inductor of Tri-phase Current). Izvestiya AN Latv. SSR, Seriya Fizicheskikh i Tekhnicheskikh Nauk, 1, 1965.
9. Mikel'son, Yu.Ya. Provodyashchiy sloy v begushchem electromagnitnom pole dvukhstoronnego nesimmetrichnogo induktora (Conductive Layer in the Traveling Electromagnetic Field of a Two-Sided Non-Symmetrical Inductor). Izvestiya AN Latv. SSR, Seriya Fizicheskikh i Tekhnicheskikh Nauk, 2, 1965.
10. Valdmanis, Ya.Ya., Liyelpeter, Ya.Ya., Mikel'son, Yu.Ya. Vliyaniye vysshikh prostranstvennykh garmonik polya na electrodinamicheskiye sily i dzhoulevy poteri v provodyashchey polose, dvizhushcheysya v begushchem magnitnom pole (Influence of Higher Spatial Field Harmonics

upon Electrodynamic Forces and Joule Heat Losses in a Conductive Band Moving in a Traveling Magnetic Field). Izvestiya AN Latv. SSR, Seriya Fizicheskikh i Tekhnicheskikh Nauk, 6, 1965.

11. Mikel'son, Yu.Ya. Uchet provodyashchikh stenok kanala induktsionnoy MGD mashiny (Computation of Conductive Channel Walls of an Induction MHD Machine). Ibid, 3, 1966.
12. Postnikov, I.M. Vikhrevyye toki v sinkhronnykh i asinchronnykh mashinakh s massivnym rotorom (Eddy Currents in Synchronous and Asynchronous Machines with a Heavy Rotor). Electrichestvo, 10, 1958.
13. Carter, F.W. Air-Gap Induction. Electr. World and Engng., p. 884, 1901.
14. Coe, R.T., Taylor, H.W. Some Problems in Electrical Machine Design Involving Eliptic Functions. Phil. Mag. and J. of Sci., 6, p. 100, 1928.
15. Vol'dek, A.I. Vliyaniye neravnomernosti vozdushnogo zazora na mannitnoye pole asinchronnoy mashiny (Influence of a Non-Uniform Air Gap upon the Magnetic Field of an Asynchronous Machine). Elektrichestvo, 12, 1951.
16. Freeman, E.M. The Calculation of Harmonics, Due to Slotting in the Flux Density Waveform of a Dynamoelectric Machine. Proc. IEE, Part C, p. 580, 1962.
17. Pecheritsa, S.P. Raschet magnitnykh poley v asinchronnom dvigatele s uchetom zubchatogo stroyeniya statora i rotora (Calculation of Magnetic Fields in an Asynchronous Engine with Allowance for a Serrated Stator and Rotor Construction). Elektrichestvo, 3, 1965.
18. Mikel'son, Yu.Ya., Sermons, G.Ya. Vliyaniye zubchatoy poverkhnosti induktora na raspredeleniye elektromagnitnogo polya v provodyashchey polose (Influence of a Serrated Inductor Surface upon the Electromagnetic Field Distribution in a Conductive Band). Izvestiya AN Latv. SSR, Seriya Fizicheskikh i Tekhnicheskikh Nauk, 1, 1966.

TRANSVERSE EDGE EFFECT IN PLANE INDUCTION MAGNETOHYDRODYNAMIC MACHINES /63

A. Ya. Vilnitis

1. Formulation of the Problem

In the general case, a plane induction magnetohydrodynamic machine (pump, generator) consists of two plane stators (inductors) with m-phase winding, a plane channel with molten metal, and a thermal-insulation channel. Figure 1 presents a sketch of a plane induction pump (1 - molten metal in the pump channel; 2 - plane pump channel; 3 - diffuser; 4 - thermal-insulation channel; 5 - pump inductor; 6 - framework of the magnetic circuit; 7 - projection of the magnetic circuit; 8 - groove of the magnetic circuit; 9 - grooved insulation; 10 - inductor winding; 11 - front portion of winding).

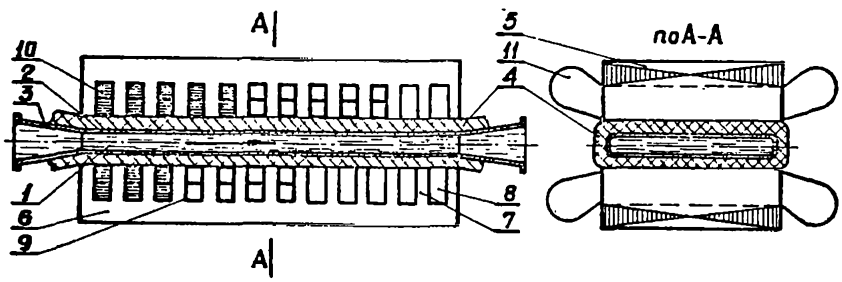

Figure 1

The currents in the windings of the inductors flow in the transverse direction, forming a traveling wave. The induced electromotive force proceeds in the same direction, which also forms a sinusoidal wave, which travels along the inductor. The sign of the electromotive force changes to the opposite sign (half of the wave length) along the pole division τ. The lines of the induced currents must be closed and -- in view of the limited width of the pump channel -- must be forced within the limits of the pole division, and closed in the longitudinal direction, in which there is no supporting electromotive force. This fact leads to attenuation of the induced curr- /64 ents. If the fact is taken into account that only the transverse current component (in interaction with the magnetic induction component which is normal to the channel plane) produces the effective force (in the longitudinal direction), then the so-called transverse edge effect becomes clear: the smaller the ratio between the channel width and the pole division, the smaller the section of the closed current line over which the supporting current of the electromotive force has an influence, and consequently the weaker is this current and the smaller the effective force for one and the same magnetic induction.

It must be immediately pointed out that this is an extremely simplified

picture. The field of the induced currents is frequently comparable with the primary field of the inductors, which causes the corresponding re-distribution of these currents. The channel thickness of the MHD machine also influences the current distribution over its width. As a result, it is far from simple to make a quantitative determination of the transverse edge effect, and it requires very detailed research.

The theory of induction MHD machines finds its source in the theory of electric machines, and is today based to a significant extent on a system of concepts developed by the latter theory. Therefore, we may draw a certain analogy between problems of electric machine theory and problems involved in the theory of induction MHD machines. If the theory of electric machines is regarded as one of the branches of Maxwell electrodynamics, this theory may be called the electrodynamics of linear circuits with currents -- currents which are closely connected with a system of conductors of previously specified directions, which are linear for the most part -- i.e., they have a zero cross-section area. The entire group of specific concepts and research methods provides a good orientation for the processes occurring in the electric machines which are used most extensively at the present time -- transformers, electric engines, and generators. The theory of electric machines -- engineering science -- and the requirement for computational simplicity which it entails plays a primary role, frequently moving a mathematically precise description of the process to second place.

If a heavy conductor or an engine containing a heavy rotor is placed in the groove of an electric machine, the electrodynamics of the linear circuits may become weak, and field theory must come to its assistance. Since we may assume that the direction of the currents in the system is known, no particular difficulties arise. This is the large rotor of infinite length, the cylindrical inductor of the traveling field, or the plane inductor of infinite width. Maxwell equations may be solved by tabulated functions, and we obtain a comparatively favorable picture of the one-dimensional skin-effect with respect to the radius of the rotor or the thickness of the plane /65 channel in pure form. Nevertheless, care must be taken in this comparatively simple case, when trying to express the results obtained in the language of the theory of electric circuits, which the theory of electric machines completely utilizes [see, for example, (Ref. 1, page 447)]. In the general case of conductors having finite diemensions, it is difficult to introduce the concept of resistance and inductance in an un-ambiguous and rational manner.

The following factor considerably complicates the problem of an engine with a large rotor having finite length or of a plane inductor having finite width: it is necessary to find not only the density of the current in the rotor (or in the channel of the plane pump, respectively) but also the current direction, which changes from point to point. The rotor in the engine is a solid body. In order to draw the corresponding analogy, it is necessary to construct a solid plate having infinite length, but finite width, between the plane inductors of the pump. Such a problem may be solved by the methods of mathematical physics. The theory of heat conductivity, the theory of elasticity, the theory of wave guides, etc. extensively employ trigonometric series -- which is a specialized method of mathematical physics

for solving partial differential equations in rectangular regions (for example, in exactly the same way that Bessel functions are characteristic for cylindrical regions) in these branches of mathematical physics.

This is the form of the problem regarding the transverse edge effect, in which we may today speak of its exact solution. The value of the exact solution thus obtained should not be over-estimated, since by no means all of the problem of the transverse edge effect in induction MHD machines has been solved. A very daring simplification of the problem is based on the assumption that a solid plate is placed between the inductors, and molten metal is the working medium of the MHD machine. When searching for technical solutions of the entire magnetohydrodynamic problem, it is very tempting to employ the comparatively simple solution for the solid plate. The trigonometric series thus obtained are orthogonal. Therefore, if we determine the averaged density of the body forces

$$\bar{\mathbf{f}}=\frac{1}{V}\int_V[\mathbf{jB}]\,dV, \tag{1}$$

$\bar{\mathbf{f}}$ is obtained in the form of a unary series, and not a binary series, if the current density $\mathbf{j}$ and the magnetic induction $\mathbf{B}$ are given by unary series. It is obtained in the form of a binary series, and not a quadruple series, if $\mathbf{j}$ and $\mathbf{B}$ are given by binary series. It is also possible to substitute the /66 initial unperturbed field $\mathbf{B}_0$ (which would occur in the absence of a conductive plate) in (1) and not the total field $\mathbf{B}$, if this simplifies the computations [see (Ref.2,p.186)]. However, maximum care must be observed. For a solid body (1) has limited application. For a liquid working medium, there is no guarantee that the pressure developed by the pump will not differ significantly from that computed on the basis of (1). In particular, this pertains to each type of deviation from the mean parameters of the machine.

In our opinion, averaging is a very risky procedure -- integration over volume. The dependence

$$\mathbf{f}=[\mathbf{jB}] \tag{2}$$

with $\mathbf{j}$ and $\mathbf{B}$ computed for a solid medium, is of value for a liquid in many cases, whereas the integral characteristics $\bar{\mathbf{f}}$ does not take several important factors into account. Therefore, it is valid to be interested in the total distribution of $\mathbf{j}$ and $\mathbf{B}$ both in theory, and in structural practice.

In every report which has been published on the transverse edge effect, it is assumed that the molten metal moves like a solid body. For purposes of clarity, we shall consider the edge effect in induction pumps, although the main results may be extended to the case of the generator regime.

In more recent studies (Ref. 3-7), the effect of attenuation of induction in the clearance and the transverse edge effect were each studied separately. Their purpose was to obtain two correction coefficients to be applied to the simple technical formula for computing pressure. Upon a closer analysis, both effects flowed together, since no separation was apparent in the phenomena. At the same time, the longitudinal edge effect,

produced by the finite length of the pump channel, previously occupied a separate position, and was studied independently of the two preceeding phenomena. At the present time, we must assume that this division is reasonable. If a purely sinusoidal (monochromatic) traveling field is produced by the inductors, then the finite dimensions of the pump channel, with respect to height and width, or of the inductor (with respect to width) are not caused by dispersion of the field wave. The amplitude and phase of induction change from point to point, but the form of the wave remains undistorted, and higher harmonics are not produced. At the same time, the periodic structure of the channel or of the inductor in the direction in which the traveling wave moves (with respect to length) inevitably produces the appearance of higher harmonics and a non-uniform non-periodic structure (inductor having a beginning or an end, or both) produces the continuous spectrum of the traveling waves. Due to this fact, mathematical procedure designed for studying the non-uniformity over the pump length differs significantly from the /67 procedure for studying the effect produced by attenuation of the field in the clearance and the transverse edge effect. It would appear that it is advantageous first to clarify the undiscovered patterns of both groups of phenomena separately. Therefore, from this point on we shall assume that the inductors and the pump channel are infinitely long.

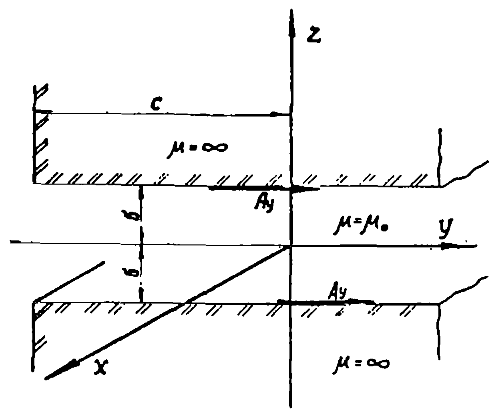

Figure 2

When examining different solutions of the problem of the transverse edge effect, we shall assume that we are dealing with a system of two inductors, the distance between which equals 2δ. These inductors carry on their surfaces (which face each other) an undistorted, traveling sinusoidal wave of the surface current (linear current loading). We shall direct the axes of the cooridinates as is shown in Figure 2. We shall let the x-axis coincide with the direction in which the traveling wave moves (along the pump), and the y-axis coincides with the direction of the surface current and of the real primary currents in the inductor windings (along the pump width). The z-axis is perpendicular to the surfaces of both inductors (along the channel width). We shall assume that the magnetic permeability of the inductors is infinite ($\mu = \infty$). In the clearance, $\mu = \mu_0$ (both in the channel with the molten metal, and outside of it). The width of the inductors is

2c (or infinity), and the channel width is 2a. We shall place the origin of the coordinates in the middle of the clearance between the inductors, and also in the middle of the inductor channels. Let the linear current loading of the inductors be

$$A_y = A_0 \cos(\omega t - \alpha x) = \operatorname{Re} \dot{A}_0 e^{i(\omega t - \alpha x)}. \tag{3}$$

Here ω is the angular frequency, and $a = \frac{\pi}{\tau}$, where τ is the pole division. All of the quantities are described in a system of SI units. If we stipulate the condition that the wave phase does not change /68

$$\omega t - \alpha x = \text{const},$$

we obtain the wave propagation velocity

$$v_n = \frac{dx}{dt} = \frac{\omega}{\alpha}. \tag{4}$$

It is understood that the directions A_y coincide for both inductors for one and the same x. The following boundary conditions hold on the inductor surfaces (in the case of $z = \pm \delta$) (no matter whether induction currents develop in the system or not, and independently of the degree to which the intra-inductor (working) clearance is filled by the current-conducting medium);

$$\begin{aligned} &\text{for } z = \delta \quad \dot{H}_x = -A_0, \\ &\text{for } z = -\delta \quad \dot{H}_x = A_0. \end{aligned} \tag{5}$$

We shall try to find the solution in a complex form, assuming the presence of a phase factor $e^{i(\omega t - \alpha x)}$ everywhere. The amplitude of this phase factor equals unity (i is the imaginary unit). Let us examine the complex amplitudes **H**, **B**, **j** and **E**, which we shall designate by a point above the appropriate letter. Then the instantaneous value of the physical quantity will equal the real part of the complex quantity

$$\mathbf{B} = \operatorname{Re} \dot{\mathbf{B}} e^{i(\omega t - \alpha x)}, \tag{6}$$

Its amplitude will equal the modulus of the complex amplitude:

$$B_k = |\dot{B}_k| \quad (k = x, y, z), \tag{7}$$

and the phase shift will equal the argument of the complex amplitude:

$$\varphi(B_k) = \arg \dot{B}_k. \tag{8}$$

We shall employ the phase of the surface current of the inductors as the reference, zero phase -- i.e., we shall assume that the complex amplitude $\dot{A}_0$ in (3) is in fact a real quantity. As long as we perform the linear operations with the components of the fields and currents, there is no necessity of dividing the real parts from the imaginary parts in the complex amplitudes. It is another matter if it is necessary to perform non-linear operations both

in formulas (1) and (2). We must then keep the fact in mind that it is /69
only possible to multiply the real physical quantities -- i.e., the real parts of type (6) complex quantities. If we are only interested in the magnitude of the product averaged over time (period), it may be computed more simply if no distinction is drawn between the real parts of the complex quantities:

$$\tilde{\mathbf{f}}=\frac{1}{2}\operatorname{Re}[\mathbf{j}\dot{\mathbf{B}}^*] \tag{9}$$

[see (Ref. 2, page 243)], where the asterisk (*) designates the complex conjugate.

2. Infinitely Wide Model

Before examining works on the transverse edge effect, let us write the well known solutions for infinitely wide systems.

1. The field in the clearance between two infinitely wide and infinitly long inductors, filled by a non-conducting medium, is

$$\begin{gathered}\dot{H}_x=\frac{\dot{H}_{x0}\operatorname{sh}\alpha z}{\operatorname{sh}\alpha\delta}=\frac{\dot{H}_{z0}\operatorname{sh}\alpha z}{i\operatorname{ch}\alpha\delta},\\ \dot{H}_z=\frac{i\dot{H}_{x0}\operatorname{ch}\alpha z}{\operatorname{sh}\alpha\delta}=\frac{\dot{H}_{z0}\operatorname{ch}\alpha z}{\operatorname{ch}\alpha\delta},\\ \dot{E}_y=\frac{\omega}{\alpha}\dot{B}_z=\frac{\mu_0\omega}{\alpha}\dot{H}_z,\\ \dot{H}_y=\dot{E}_x=\dot{E}_z=0.\end{gathered} \tag{10}$$

According to (5) we have

$$\dot{H}_{x0}=-A_0;\quad \dot{H}_{z0}=\frac{i\dot{H}_{x0}}{\operatorname{th}\alpha\delta}. \tag{11}$$

2. The field in the clearance which is uniformly filled with a medium having the conductivity σ is

$$\begin{gathered}\dot{H}_x=\frac{\dot{H}_{x0}\operatorname{sh}\beta z}{\operatorname{sh}\beta\delta}=\frac{\beta\dot{H}_{z0}\operatorname{sh}\beta z}{i\alpha\operatorname{ch}\beta\delta},\\ \dot{H}_z=\frac{i\alpha\dot{H}_{x0}\operatorname{ch}\beta z}{\beta\operatorname{sh}\beta\delta}=\frac{\dot{H}_{z0}\operatorname{ch}\beta z}{\operatorname{ch}\beta\delta},\\ \dot{E}_y=\frac{\omega}{\alpha}\dot{B}_z=\frac{\mu_0\omega}{\alpha}\dot{H}_z,\quad j_y=\sigma\dot{E}_y,\\ \dot{H}_y=j_x=j_z=0.\end{gathered} \tag{12}$$

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Here we have

$$\beta^2=\alpha^2+i\mu_0\sigma\omega; \tag{13}$$

$$\dot{H}_{x0} = -A_0, \quad \text{но} \quad \dot{H}_{z0} = \frac{i\alpha \dot{H}_{x0}}{\beta \operatorname{th} \beta\delta}. \tag{14}$$

In formulas (12) -- (14) (the same pertains to the following formulas) we may divide the real and imaginary part, before commencing the numerical computation which, however, leads to cumbersome expressions. On the other hand, we may perform the entire computation from beginning to end with complex numbers, as is sometimes done in the theory of circuits. It is our opinion that the second method is preferable.

Let us study the limiting case $\delta \ll \tau$, or $\alpha\delta \ll \pi$. Then th $a\delta \to 0$, and according to (11), $\dot{H}_{z0} \gg \dot{H}_{x0}$. But $\dot{H}_{z0} = \frac{\alpha}{\mu_0\omega} \dot{E}_{y0}$, and $\dot{E}_{y0}$ play the role of counter-electromotive force per unit of inductor width. There is no reason to assume that it strives to infinity, just like the applied voltage. For a sufficiently narrow clearance, $\dot{H}_x \to 0$ and $\dot{A}_y \to 0$, and $\dot{H}_z = \frac{\alpha}{\mu_0\omega} \dot{E}_y$ does not strive to infinity. In addition, we have

$$\frac{\partial \dot{H}_z}{\partial z} = \frac{\alpha \dot{H}_{z0} \operatorname{sh} \alpha z}{\operatorname{ch} \alpha\delta} \to 0, \quad \frac{\partial \dot{E}_y}{\partial z} \to 0,$$

but

$$\frac{\partial \dot{H}_x}{\partial z} = \frac{\alpha \dot{H}_{z0} \operatorname{ch} \alpha z}{i \operatorname{ch} \alpha\delta} \to \frac{\alpha}{i} \dot{H}_{z0}, \tag{15}$$

and not to zero. Thus, the longitudinal field component strives to zero for a sufficiently small clearance. This does not occur for its derivative with respect to z, which strives to the finite limit (15). In its turn, the derivative (with respect to z) of $\dot{H}_z$ strives to zero; the normal field component becomes independent of z. In order that $\dot{H}_x \to 0$ may be fulfilled in the case of a clearance filled by a conductive medium, we must set $|\beta\delta| \ll \pi$, which means $\alpha\delta \ll \pi$ and $\sqrt{\mu_0\sigma\omega\delta} \ll \pi$ at the same time. The latter is fulfilled in all induction pumps employed in practice (if $\mu_0\sigma\omega \gg a^2$, such a pump is not feasible due to an exaggerated skin-effect). If we retain the linear current loading of the inductors (more correctly, the ratio of the surface current to the clearance thickness $\lim_{\delta\to 0} \frac{A_0}{\delta}$, since the current itself strives to zero), we then obtain the following when there is no conductor in the clearance /71

$$\lim_{\delta\to 0} \frac{\partial \dot{H}_x}{\partial z} = \frac{\alpha}{\alpha} \lim_{\delta\to 0} \frac{\dot{H}_{x0}}{\delta} = -\lim_{\delta\to 0} \frac{A_0}{\delta}, \tag{16}$$

and we obtain the following when a conductor is present

$$\lim_{\delta\to 0} \frac{\partial \dot{H}_x}{\partial z} = \frac{\beta}{\beta} \lim_{\delta\to 0} \frac{\dot{H}_{x0}}{\delta} = -\lim_{\delta\to 0} \frac{A_0}{\delta},$$

i.e., we obtain one and the same result in both cases.

Formulas (10) - (14) make it possible to determine the averaged force per unit of surface of solid plate cross-section between the inductors, which we shall designate as the pressure of the induction pump:

$$p=\frac{L}{2}\mathrm{Re}\frac{1}{2a}\int_{-a}^{a}dy\frac{1}{2\delta}\int_{-\delta}^{\delta}dz\,(j_y\dot{B}_z'^{*}). \tag{17}$$

We would like to add a few words to (17). We obtain p from (1) by multiplying $\bar{f}_x$ by the pump length L and averaging over time, disregarding the longitidunal edge effects. For whatever reason, the product $j_z\dot{B}_y{}^*$ always equals zero. Also $\tilde{f}_y = \tilde{f}_z = 0$. We employ $\dot{B}'_z$ in (17) to designate the unperturbed field (the field in a hollow clearance), according to the condition mentioned above which simplifies the computation. If there is no dependence on y, we then have $\frac{1}{2a}\int_{-a}^{a} dy = 1$. The same holds true for z.

The pressure is calculated very simply for an infinitely thin clearance between infinitely wide inductors if the medium filling the clearance has such a low conductivity σ that the field of the induced currents is much weaker than the primary field ($\mu_0\sigma\omega \ll \alpha^2$). We may then employ formulas (10), in which $\delta \to 0$, and $j_y = \sigma\dot{E}_y$ (although it is small, σ is entirely different from zero). In this case (17) yields

$$p_0=\frac{\sigma\omega\,|\dot{B}'_{z0}|^2 L}{2\alpha}. \tag{18}$$

3. The Theory Advanced by A. I. Vol'dek /72

Let us now turn immediately to the problem of the transverse edge effect. In chronological terms, the first analytical study deals with the transverse edge effect "in pure form" -- i.e., when there is no attenuation of the field in the clearance -- which is achieved in the limit by the thin clearance between the inductors. A. I. Vol'dek conducted this study (Ref. 3). In view of the importance of this solution, we shall present it in its entirety below, together with certain comments. A similar problem was investigated in (Ref. 4) for rotational machines.

A. I. Vol'dek divided the field in the clearance between the inductors into a primary and secondary field. The primary field is the field in the clearance filled by a non-conductive medium. (We shall designate it by **H**' and **E**'). In view of the fact that **j**' = 0, we must have rot **H**' = 0. The secondary field is the field of the induced currents (we shall designate it by **H**" and **E**"). The total field equals the sum **H** = **H**' + **H**", **E**, respectively. The Maxwell equations

$$\text{rot}\,\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{19}$$

$$\text{rot}\,\mathbf{H} = \mathbf{j} \tag{20}$$

must be fulfilled for the total field. In view of rot $\mathbf{H}' = 0$, we obtain the following for the secondary field

$$\text{rot}\,\mathbf{E}'' = -\frac{\partial \mathbf{B}''}{\partial t}, \tag{21}$$

$$\text{rot}\,\mathbf{H}'' = \mathbf{j} = \sigma(\mathbf{E}' + \mathbf{E}''). \tag{22}$$

The reason for this division lies in the fact that $\mathbf{E}'$ is assumed to be a known function. In coȯrdinate notation, (21) and (22) yield

$$\frac{\partial \dot{E}_z''}{\partial y} - \frac{\partial \dot{E}_y''}{\partial z} = -i\omega \dot{B}_x'', \quad \frac{\partial \dot{H}_z''}{\partial y} - \frac{\partial \dot{H}_y''}{\partial z} = \sigma(\dot{E}_x' + \dot{E}_x''),$$

$$\frac{\partial \dot{E}_x''}{\partial z} + i\alpha \dot{E}_z'' = -i\omega \dot{B}_y'', \quad \frac{\partial \dot{H}_x''}{\partial z} + i\alpha \dot{H}_z'' = \sigma(\dot{E}_y' + \dot{E}_y''),$$

$$i\alpha \dot{E}_y'' + \frac{\partial \dot{E}_x''}{\partial y} = i\omega \dot{B}_z'', \quad -i\alpha \dot{H}_y'' - \frac{\partial \dot{H}_x''}{\partial y} = j_z.$$

We must simplify the system (21) - (22), based on the physical conditions in an extremely thin plate between the inductors. It is apparent that we must have $j_z = 0$ in this plate -- i.e., the current lines lie on one plane. We must also assume that $\dot{E}_z = 0$, since the plate has the same conductivity σ along the z-axis. For infinitely wide inductors $\dot{E}'_x = \dot{E}'_z = 0$. This effect may be /73 expected for an extremely thin clearance between inductors of finite thickness, so long as regions are examined which are not too close to the inductor edges. We then have $\frac{\partial E'_y}{\partial y} = 0$ and $E''_z = 0$. The occurrence of E''_x is caused by the finite, wide plate. Since rot $\mathbf{A} \perp \mathbf{A}$, then -- according to (22) -- $\mathbf{H}'' \perp \mathbf{j}$, which may be satisfied in the case of $\mathbf{H}'' = e_z H''_z$, although this is not the only possibility. However, preference should be given to this, based on the following considerations: if we retain the surface current A_y when introducing a conductive plate, then for $\dot{H}''_x$ we obtain zero boundary conditions [see (5)]. Thus, we also have $\frac{\partial \dot{H}''_x}{\partial 2} = 0$ [see (16)]. Even when A_y changes, we may write

$$\left.\frac{\partial \dot{H}''_x}{\partial y}\right|_{z = \pm\delta} = 0,$$ since volume currents have no influence on the boundary conditions, and $\frac{\partial A_y}{\partial y} = 0$ always holds. Therefore, it is natural to assume that $\dot{H}''_x = 0$. We also have zero boundary conditions for $\dot{H}''_y$, since $\dot{A}_x = 0$. We must

thus assume that $\dot{H}''_y = 0$. We may express this most clearly in the following way. The lines of the induced current field are closed between the inductors along the shortest path -- i.e., perpendicular to the plane of the clearance. The zero boundary conditions for $\dot{H}''_x$ and $\dot{H}''_y$ designate the perpendicular nature of the lines to the inductors when they enter them. However, we than have

$$\frac{\partial \dot{E}''_x}{\partial z} = \frac{\partial \dot{E}''_y}{\partial z} = 0, \text{from } \operatorname{div} \mathbf{H}'' = 0 \text{ we have } \frac{\partial \dot{H}''_z}{\partial z} = 0,$$

and equations (21) (22) yield the system which is used as the initial system of A. I. Vol'dek:

$$\begin{aligned} i\alpha \dot{E}''_y + \frac{\partial \dot{E}''_x}{\partial y} &= i\mu_0 \omega \dot{H}''_z; \\ \frac{\partial \dot{H}''_z}{\partial y} &= \sigma \dot{E}''_x; \\ i\alpha \dot{H}''_z &= \sigma \dot{E}''_y + \sigma \dot{E}'_y. \end{aligned} \tag{23}$$

Excluding $\dot{E}''_x$ and $\dot{H}''_z$ and employing $\frac{\partial \dot{E}'_y}{\partial y} = 0$, we obtain the following from (23)

$$\frac{\partial^2 \dot{E}''_y}{\partial y^2} = \beta^2 \dot{E}''_y + i\mu_0 \sigma \omega \dot{E}'_y, \tag{24}$$

where β is defined according to (13). (24) has the solution /74

$$\dot{E}''_y = A_1 \operatorname{ch} \beta y + A_2 \operatorname{sh} \beta y - \frac{i\mu_0 \sigma \omega}{\beta^2} \dot{E}'_y,$$

and then the current is

$$j_y = \sigma(\dot{E}''_y + \dot{E}'_y) = \sigma\left(A_1 \operatorname{ch} \beta y + A_2 \operatorname{sh} \beta y + \frac{\alpha^2}{\beta^2} \dot{E}'_y\right).$$

The natural boundary conditions are $j_y = 0$ in the case of $y = \pm a$ [A.I. Vol'dek stipulates $\dot{H}''_z = 0$ in the case of $y = \pm a$ which is one and the same for the system (23), see (23) -- third], from which it follows that $A_2 = 0$, since j_y must be an even function of y and $A_1 = -\frac{a^2 \dot{E}'_y}{\beta^2 \operatorname{ch} \beta a}$. Correspondingly, we have

$$\begin{aligned} j_y &= \frac{\alpha^2}{\beta^2} \sigma \dot{E}'_y \left(1 - \frac{\operatorname{ch} \beta y}{\operatorname{ch} \beta a}\right); \\ \dot{H}''_z &= \frac{\alpha \sigma \dot{E}'_y}{i\beta^2} \left(1 - \frac{\operatorname{ch} \beta y}{\operatorname{ch} \beta a}\right); \\ j_x &= \frac{i\alpha \sigma \dot{E}'_y}{\beta} \cdot \frac{\operatorname{sh} \beta y}{\operatorname{ch} \beta a}. \end{aligned} \tag{25}$$

We feel it is necessary to add certain remarks to the solution (25). As has already been indicated, we may assume that $\dot{E}'_x = \dot{E}'_z = \dot{B}'_y = 0$ for the

regions (which are central with respect to the width) of the clearance between the inductors. In this case, solution (25) must be explained by the assumption that the conductive plate is narrower than the inductors. Nevertheless, A. I. Vol'dek extends the solution (25) both to the case a = c (plate having the same width as the inductors) and to the case a>c (plate wider than the inductors). This is based on the fact that $\dot{H}'_z$ decreases sharply at the outermost edge of the inductor, and 2c barely changes over the entire area, so that the $\dot{H}'_z$, which is average with respect to the inductor width, differs little from maximum $\dot{H}'_z$. This latter fact is indisputable, but we would like to note that $\overline{\partial \dot{H}'_z} \to \pm \infty$ close to the inductor edges, and consequently $\frac{\partial \dot{H}'_y}{\partial z} \to \pm \infty$ and $\frac{\partial^2 \dot{H}'^{\,\partial y}_x}{\partial y \partial z} \to \mp i\alpha\, \infty$. We also have $\frac{\partial \dot{E}'_y}{\partial y} \to \pm \infty$, from which we have $i\alpha \dot{E}'_x - \frac{\partial \dot{E}'_z}{\partial z} \to \mp \infty$. Unfortunately, no conclusions may be drawn regarding the quantity $\frac{\partial \dot{E}'_y}{\partial y}$, if we assume that $\frac{\partial \dot{H}'_z}{\partial y}$ is given. No conclusions may be drawn with respect to the quantity /75
$\dot{E}'_x$ either. In order to do this, we must examine the solution in the entire region of the field propagation.

After these brief comments, let us turn to the solution (25). According to (9), we have

$$\tilde{f}_x = \frac{1}{2}\,\mathrm{Re}\,[j_y(\dot{B}_z'^{*} + \dot{B}_z''^{*})].$$

However, it may be seen from (25) that j_y and $\dot{H}''_z$ differ by the factor $\frac{\alpha}{i}$ -- i.e., they are shifted with respect to phase by 90°. Consequently, $j_y \dot{B}''_z{}^*$ cannot provide the resulting force. According to (17), the pressure is

$$p = \frac{\sigma\omega\,|\dot{B}^2_{z0}|\,L}{2\,\alpha}\cdot\frac{1}{2a}\int_{-a}^{a}\mathrm{Re}\frac{\alpha^2}{\beta^2}\left(1-\frac{\mathrm{ch}\,\beta y}{\mathrm{ch}\,\beta a}\right)dy = \tag{26}$$

$$= \frac{\sigma\omega\,|\dot{B}^2_{z0}|\,L}{2\,\alpha}\,\mathrm{Re}\frac{\alpha^2}{\beta^2}\left(1-\frac{\mathrm{th}\,\beta a}{\beta a}\right).$$

A. I. Vol'dek recommends that the pressure obtained (26) be expressed by the pressure p_0 (18) in the clearance filled by a medium with low conductivity, and introduces the attenuation coefficient k^{I}_{oc}:

$$p = k^{I}_{oc}\, p_0. \tag{27}$$

This method has certain advantages for technical designs of pumps. The corresponding attenuation coefficient ($a \neq \infty$, $\sigma \neq 0$, $\delta \to 0$) is obtained as equal to

$$k^{I}_{oc} = \mathrm{Re}\,\frac{\alpha^2}{\beta^2}\left(1 - \frac{\mathrm{th}\,\beta a}{\beta a}\right). \tag{28}$$

It should be noted that k^{I}_{oc} is obtained for constant linear current loading of the inductors (more accurately, for constant $\lim\limits_{\delta \to 0} \frac{\dot{A}_y}{\delta}$, since the current loading $\dot{A}_y$ and the clearance thickness 2δ strive to zero). In other words, an operation is performed of inserting a highly conductive plate of finite width in the extremely narrow clearance (which was hollow before this) between two infinitely wide inductors, maintaining the current in the inductor windings, and not the supply voltage, which must thus be reduced since the total winding resistance has decreased.

In the case of $a \to \infty$ -- i.e., when the entire, infinitely wide clearance is filled quite well by a conductive medium (retaining $\lim\limits_{\delta \to 0} \frac{A_y}{\delta}$) -- we have /76

$$k_{oc}^{I} \to k_{oc}^{II} = \mathrm{Re}\,\frac{\alpha^2}{\beta^2} = \frac{\alpha^4}{|\beta^4|}. \tag{29}$$

From our point of view, the following expression is assumed for k^{I}_{oc}, which is obtained by expansion of th βa in elementary fractions:

$$k^{I}_{oc} = \frac{2\alpha^2}{a^2}\sum_{n=1}^{\infty}\frac{1}{\varkappa_n^2(\alpha^2+\varkappa_n^2)}\cdot\frac{1}{1+\frac{\mu_0^2\sigma^2\omega^2}{(\alpha^2+\varkappa_n^2)^2}}, \tag{30}$$

where

$$\varkappa_n = \frac{(2n-1)\pi}{2a}. \tag{31}$$

Under the condition that $\mu_0\sigma\omega \ll \alpha^2$ ($a \neq \infty$, $\sigma \to 0$, $\delta \to 0$), we obtain the following from (28)

$$k_{oc}^{III} = 1 - \frac{\mathrm{th}\,\alpha a}{\alpha a}, \tag{32}$$

which may be readily determined according to a table of functions. The difference $k_{oc}^{III} - k_{oc}^{I}$ may be written in the form of a rapidly converging series:

$$k_{oc}^{III} - k_{oc}^{I} = \frac{2\,\alpha^2\mu_0^2\sigma^2\omega^2}{a^2}\sum_{n=1}^{\infty}\frac{1}{\varkappa_n^2(\alpha^2+\varkappa_n^2)^3}\cdot\frac{1}{1+\frac{\mu_0^2\sigma^2\omega^2}{(\alpha^2+\varkappa_n^2)^2}}. \tag{33}$$

In the case when a conductive plate thickness does not occupy the entire clearance 2δ, but only part of it, 2Δ, so that $\delta > \Delta$, A. I. Vol'dek adheres to the previous hypothesis $\dot{H}_x = 0$. In this case, the magnetic resistance to the flux produced by the induced currents increases by a factor of δ/Δ, which is equivalent to a decrease in the magnetic permeability (or conductivity) of the medium by as much as $\left(\mu_0 \to \frac{\mu_0\Delta}{\delta}\right)$.

It is a very simple process to pass from a plate at rest to a moving plate: it is sufficient to multiply the cyclic frequency of the field ω by the slipping s ($\omega \to \omega s$, where $s = 1 - \frac{v}{v_f}$; v -- velocity of the plate; v_f -- velocity of the field) [see (4)].

As was already pointed out, $k_{oc}{}^{I}$ may be derived for a constant linear current loading of the inductors. If we have a plate with high conductivity or if we are operating at a high frequency, then $\mu_0\sigma\omega$ becomes larger, or even /77 much larger than, α^2. In these cases, $k_{oc}{}^{I}$ has very small numerical values. According to the terminology employed, we thus have a strong armature reaction (which is expressed as a drop in the total winding resistance). If it is not balanced -- i.e., if the previous linear current loading of the inductors is maintained -- then the effectiveness of the pump greatly decreases. However, in practice (if only the temperature regime of the winding permits it) it is frequently not the current which remains constant, but the supply voltage, and the armature reaction is balanced. It is difficult to make a precise calculation of the effect of a constant supply voltage, due to the absence of a single theory for a non-magnetic clearance and for the inductor itself: the real inductor is replaced by a ferromagnetic body with a surface current loading for which there is no active inner resistance, nor inductive inner resistance. If we retain this idealization, we may only speak of counter-electromotive force to the supply voltage, caused by the variable magnetic field in the clearance. The constant supply voltage is then equivalent to a constant magnetic flux $\dot{\Phi}_\tau$ for the pole division. In infinitely wide systems, a constant $\dot{\Phi}_\tau$ is equivalent to a constant $\dot{B}_{z0}$. In systems of finite width, there is no point in speaking of a constant $\dot{B}_{z0} = \dot{B}'_{z0} + \dot{B}''_{z0}$, since $\dot{B}''_{z0}$ depends on y [see (25)]. According to this, we may introduce the attenuation coefficient for a constant supply voltage (in the sense of a constant magnetic flux $\dot{\Phi}_\tau$), i.e., with compensation for the armature reaction. As has already been pointed out, the theory of A. I. Vol'dek encompasses the tacit assumption that the inductors are wider than the channel (to a sufficient extent). Let us clarify this assumption: c > a. It is assumed that the winding is of a diametrical step. We then have

$$\dot{U}^0 = i\omega\dot{\Phi}_\tau{}^0 = \frac{4\omega c}{\alpha}\dot{B}_{z0}{}^0,$$

$$\dot{U} = i\omega\dot{\Phi}_\tau = \frac{4\omega c}{\alpha}\dot{B}_{z0}' + i\omega\int_0^\tau e^{-i\alpha x}dx\int_{-a}^{a}\dot{B}_{z0}''\,dy =$$

$$= \frac{4\omega}{\alpha}\dot{B}_{z0}'\left[c + \frac{\mu_0\sigma\omega\, a}{i\beta^2}\left(1 - \frac{\operatorname{th}\beta a}{\beta a}\right)\right];$$

and stipulating that $\dot{U} = \dot{U}^0$, we obtain

$$\dot{B}_{z0}' = \frac{\dot{B}_{z0}{}^0}{1 + \frac{\mu_0\sigma\omega}{i\beta^2}\cdot\frac{a}{c}\left(1 - \frac{\operatorname{th}\beta a}{\beta a}\right)}$$

and /78

$$k_{oc}^{IV} = \mathrm{Re}\,\frac{\alpha^2\left(1-\frac{\mathrm{th}\,\beta a}{\beta a}\right)}{\beta^2 - i\mu_0\sigma\omega\frac{a}{c}\left(1-\frac{\mathrm{th}\,\beta a}{\beta a}\right)}. \tag{34}$$

It may be readily seen that $\lim\limits_{a=c\to\infty} k_{oc}^{IV} = 1$ -- i.e., if the supply voltage is maintained, the armature reaction in an infinitely wide plate is completely balanced.

4. Other Theories with Plane-Parallel Field in the Clearance of the Machine

L. V. Nitsetskiy (Ref. 5) has formulated a principle for drawing up a model of certain vortex fields in an electrolytic bath and on electroconductive paper. In particular, this principle may be applied to formulate a model of the transverse edge effect in an extremely thin plate. It has been verified by the author by comparison with the results derived from performing the calculation according to formula (32). The agreement was satisfactory.

L. Ya. Ulmanis (Ref. 6) employed an analytical method to obtain the following attenuation coefficient for an extremely thin finite width of the plate

$$k_{oc}^{V} = \frac{|\beta^4|}{\alpha^4}\,\mathrm{Re}\frac{\alpha^2}{\beta^2}\left(1-\frac{\mathrm{th}\,\beta a}{\beta a}\right) \tag{35}$$

([Ref. 6], formula (1); the previous notation is employed). k_{oc}^{V} is determined as the ratio of the pressure in a plate of finite thickness to the pressure in an infinitely wide plate, with allowance for the armature reaction:

$$p = k_{oc}^{V} p'_0,$$

where $p'_0 = k_{oc}^{II} p_0$ is the pressure in an infinitely wide, highly conductive plate, which is chosen as the initial pressure, instead of p_0 (18). As the plate narrows from an infinite width to the width 2a, a constant current loading of the inductors $\left(\lim\limits_{\delta\to 0} \frac{\dot{A}_y}{\delta}\right)$ is maintained. L. Ya. Ulmanis has checked (28), (32) and (35) experimentally, and has provided the following experimental formula

$$k_{oc}^{VI} = \frac{1}{1+1.3\left(\frac{\tau}{a}\right)^2}. \tag{36}$$

Nothing more similar may be obtained, assuming $\sigma = 0$ in the expansion of (30) and taking the first term of the series /79

$$\lim_{\sigma\to 0}(k_{oc}^{I})_1 = \frac{\pi^2}{8}\,\frac{1}{\left[1+\left(\frac{\tau}{a}\right)^2\right]}.$$

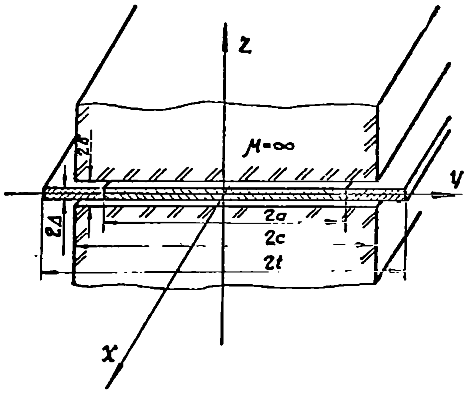

Figure 3

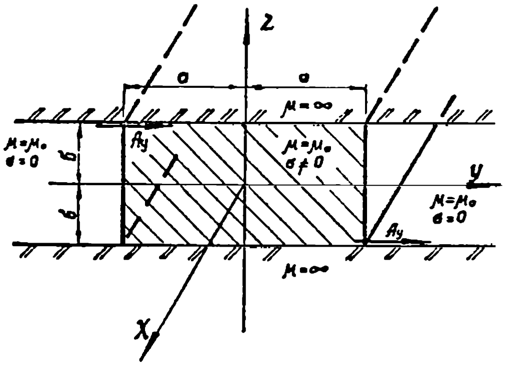

Figure 4

In an article by A. I. Vol'dek and Kh. A. Yanes (Ref. 7), the theory which was discussed for an extremely thin plate between inductors was generalized to the case of a plate with lateral short-circuiting bus-bars (Figure 3). Zone I is the plate which simulates the channel of a pump with molten metal (it is immobile here!), and the conductivity of the zone is σ. Zone II is the short-circuiting bus-bars with the conductivity σ_2. The condition $\delta > \Delta$ is taken into account by the introduction of equivalent conductivity of the zone $\sigma' = \sigma \frac{\Delta}{\delta}$, which is similar for σ_2. It is assumed that the primary magnetic field is fully concentrated in the clearance: in the case of $|y| < c$ $\dot{B}'_z$ = const, and in the case of $|y| > c$ $\dot{B}'_z = 0$ (just like $\dot{E}'_y$). Two special cases were examined in this study.

1. a = c (short-circuiting bus-bars outside of the inductors). Equation (23) holds for zone I. $\dot{E}'_y = 0$ in zone II, and the field of the induced currents is also disregarded, in view of the fact that magnetic lines pass along a non-magnetic medium. Therefore, we may formally substitute $\mu_0 = 0$ in (23). The boundary conditions are: in the case of $y = \pm t$ $\dot{H}''_z = 0$, and in the case of $y = \pm a$ $j_y|_I = j_y|_{II}$; $\dot{E}''_x|_I = \dot{E}''_x|_{II}$. In addition, it is assumed /80
that $H_z|_I$ is an even function of y. The following result is obtained:

$$\dot{H}_z''|_I = -\frac{i\alpha\sigma\dot{E}_y'}{\beta^2}\left[1-\frac{\alpha\sigma\,\mathrm{ch}\,\beta y}{\alpha\sigma\,\mathrm{ch}\,\beta c+\sigma_2\beta\,\mathrm{th}\,\alpha(t-c)\,\mathrm{sh}\,\beta c}\right];$$

$$j_y|_I = i\alpha\dot{H}_z''|_I; \tag{37}$$

$$k_{oc}^{VII} = \mathrm{Re}\frac{\alpha^2}{\beta^2}\left\{1-\frac{\alpha\sigma\,\mathrm{sh}\,\beta c}{\beta c\,[\alpha\sigma\,\mathrm{ch}\,\beta c+\beta\sigma_2\,\mathrm{th}\,\alpha(t-c)\,\mathrm{sh}\,\beta c]}\right\}.$$

Substituting $\sigma_2 = \sigma$ from (37), we may obtain the formulas for a channel which is wider than the inductors, but without short-circuiting bus-bars.

2. t = c (short-circuiting bus-bars between the inductors). In this case, we have equations (23) for both regions; the boundary conditions are the same as before. The result is

$$\begin{gathered}\dot{H}_z''|_1 = -\frac{i\alpha\sigma\dot{E}_y'}{\beta^2}\left\{1 - \right.\\ \left. -\frac{[(\sigma\beta_2^2-\sigma_2\beta^2)\,\mathrm{ch}\,\beta_2(c-a)+\sigma_2\beta^2]\,\mathrm{ch}\,\beta y}{\beta_2[\sigma_2\beta\,\mathrm{sh}\,\beta_2(c-a)\,\mathrm{sh}\,\beta a+\sigma\beta_2\,\mathrm{ch}\,\beta_2(c-a)\,\mathrm{ch}\,\beta a]}\right\},\\ j_y|_1 = i\alpha\dot{H}_z''|_1,\\ k_{oc}^{\mathrm{VIII}} = \mathrm{Re}\,\frac{\alpha^2}{\beta^2}\left\{1 - \right.\\ \left. -\frac{\mathrm{sh}\,\beta a\,[(\sigma\beta_2^2-\sigma_2\beta^2)\,\mathrm{ch}\,\beta_2(c-a)+\sigma_2\beta^2]}{\beta a\beta_2[\sigma_2\beta\,\mathrm{sh}\,\beta_2(c-a)\,\mathrm{sh}\,\beta a+\sigma\beta_2\,\mathrm{ch}\,\beta_2(c-a)\,\mathrm{ch}\,\beta a]}\right\}.\end{gathered} \tag{38}$$

Here $\beta_2^2 = \alpha^2 + i\mu_0\sigma_2\omega$. Cases are possible in which $k_{oc}^{VIII}>1$ for small values of σ and σ_2, and also a.

k_{oc}^{VII} and k_{oc}^{VIII} are introduced under the assumption that the linear current loading of the inductors is constant when conductive plates are introduced into the clearance of the system.

5. Transverse Edge Effect in a Clearance of Finite Thickness

The following stage in an investigation of the transverse edge effect in induction pumps is related to the transition from an extremely thin plate between the inductors to a plate of finite thickness. It is discussed in the publications of A. Ya. Vilnitis (Ref. 8), T. A. Veske (Ref. 9), and N. M. Okhremenko (Ref. 10, 11). In veiw of the fact that identical solutions are obtained in the studies (Ref. 8, 9), in spite of certain variations in the formulations of the problems, it is advantageous to analyze them concurrently.

The following problem is postulated in the study (Ref. 8). There are /81 two infinitely wide and long inductors with $\mu = \infty$, which carry a linear current loading in the form of a sinusoidal traveling wave of the surface current A_y (Figure 4). A conductive body having rectangular cross-section and the thickness 2δ (which equals the distance between the inductors) and the width 2a is placed in the intra-inductor space. In the remaining intra-inductor space $\sigma = 0$. It is assumed that the body is immobile, but -- as was already pointed out -- it is sufficient to multiply the cyclical frequency of the field ω by the slipping s for the transition to the mobile body. When the problem is solved, the boundary conditions $j_y = 0$ in the case of $y = \pm a$ and $j_z = 0$ in the case of $z = \pm\delta$ become apparent, in addition to the boundary condition (5). It is also necessary that the exciting action of the body may be reduced to zero at an infinite distance from this body, which is placed between the inductors -- i.e., in the case of $y \to \pm\infty$, the field between the inductors may be expressed according to (10). Finally, it is necessary that the magnetic field be constant in the case of $y = \pm a$.

The solution of the problem which is given in (Ref. 8) follows very clearly from these conditions. Two qualitative consequences are clearly apparent in this problem, which were not apparent from the very beginning:

(1) There is no transverse component of the magnetic field B_y over the entire clearance, both outside of the conductive body, and inside of it;

(2) The field outside of the conductive body is an unperturbed field not only in the case of $y \to \pm \infty$, but also for any $|y| \geqslant a$.

By analogy with the formulation of the problem given by A. I. Vol'dek for an extremely thin plate, the author (Ref. 9) first assumed that the plate was thinner than the clearance between the inductors. However, when written out the solution everywhere becomes $\delta = \Delta$ -- i.e., the solution may be written for a plate occupying the entire width of the clearance. Secondly, the author /82 did not define the problem of the inductor width more accurately, and only assumed that the primary field is described by expressions (10). This is equivalent to the assumption of infinitely wide inductors. Thirdly, it was postulated that the secondary field (the field of the induced currents) does not pass beyond the side boundaries of the conductive plate . It is not clearly apparent that this is an initial condition, but this is actually the case.

If the solutions given in (Ref. 8, 9) are reduced to one and the same notations, the fact that they are completely identical becomes readily apparent -- i.e., expressions (Ref. 8) (42) are identical to (Ref. 9) (4), and expressions (Ref. 8) (41) are identical to (Ref. 9) (5) (with allowance for the fact that the total field is given in [Ref. 8], and that the primary field (Ref. 9) (3) must be added to (Ref. 9) (4), (5). This fact apparently indicates that the problem may be solved without errors under the given premises, and that with these solutions the next stage in terms of complexity is, in a certain sense, exhausted after the solution for the extremely thin plate, given by A. I. Vol'dek. We cannot substantiate the fact that all of the qualitative patterns hidden in the above-mentioned solutions have been clarified. Their clarification is of essential importance for orientation when proceeding to the next stage of the problem in terms of complexity.

The expressions for the fields and the currents are given in six different representations in (Ref. 8) and (Ref. 9). We shall give the expressions for $\dot{B}_z$ and j_y (i.e., for the components which are of interest when computing the pressure developed by the pump) in a form which is the most advantageous for the computations:

$$\dot{B}_z=\dot{B}_{x0}\left\{\frac{i\alpha}{\beta}\cdot\frac{\operatorname{ch}\beta z}{\operatorname{sh}\beta\delta}-\frac{\mu_0\sigma\omega}{\alpha\delta\beta^2}\cdot\frac{\operatorname{ch}\beta y}{\operatorname{ch}\beta a}+\right.$$
$$\left.+\frac{2\mu_0\sigma\omega a}{\delta}\sum_{m=1}^{\infty}\frac{(-1)^{m-1}\operatorname{ch}\gamma_m y}{\gamma^2_m\gamma'^2_m\operatorname{ch}\gamma_m a}\cos\lambda_m z\right\}, \quad (39)$$

$$j_y=\dot{B}_{x0}\left\{\frac{i\sigma\omega}{\beta}\cdot\frac{\operatorname{ch}\beta z}{\operatorname{sh}\beta\delta}-\frac{i\sigma\omega}{\delta\beta^2}\cdot\frac{\operatorname{ch}\beta y}{\operatorname{ch}\beta a}+\right. \quad (40)$$

$$+\frac{2i\sigma\omega}{\delta}\sum_{m=1}^{\infty}\frac{(-1)^{m-1}\operatorname{ch}\gamma_m y}{\gamma^2_m\operatorname{ch}\gamma_m a}\cos\lambda_m z\Big\}, \tag{40}$$

where

$$\lambda_m=\frac{\pi m}{\delta};\quad \gamma_m{}^2=\beta^2+\lambda_m{}^2=\beta^2+\frac{\pi^2 m^2}{\delta^2}; \tag{41}$$

$$\gamma_m'^2=\alpha^2+\lambda_m^2, \tag{42}$$

and β is given by the expression (13). The series in (39) and (40), which cannot be summed up in tabulated functions, converge quite rapidly, primarily due to the factor $\frac{\operatorname{ch}\gamma_m y}{\operatorname{ch}\gamma_m a}$, since $|y|<a$. In the case of $y = \pm a$, the series are summed up as follows: /83

$$\begin{gathered}\sum_m\frac{(-1)^{m-1}\cos\lambda_m z}{\gamma_m^2}=\frac{1}{2\beta^2}\left(1-\frac{\beta\delta\operatorname{ch}\beta z}{\operatorname{sh}\beta\delta}\right),\\ \sum_m\frac{(-1)^{m-1}\cos\lambda_m z}{\gamma^2_m\gamma'^2_m}=\frac{1}{2i\mu_0\sigma\omega}\left\{\frac{1}{\alpha^2}\left(1-\frac{\alpha\delta\operatorname{ch}\alpha z}{\operatorname{sh}\alpha\delta}\right)-\right.\\ \left.-\frac{1}{\beta^2}\left(1-\frac{\beta\delta\operatorname{ch}\beta z}{\operatorname{sh}\beta\delta}\right)\right\}.\end{gathered} \tag{43}$$

It may be readily seen that (39) and (40) are valid both over the region, and at the boundaries of the conductive body.

It is interesting to note that the first terms in (39) and (40) coincide with the formulas (2) for an infinitely wide channel, and the second terms may be written as follows

$$-\dot{B}_{x0}\frac{\mu_0\sigma\omega}{\alpha\delta\beta^2}\cdot\frac{\operatorname{ch}\beta y}{\operatorname{ch}\beta a}=\lim_{\substack{\delta\to 0\\ a\neq\infty\\ \sigma\neq 0}}\dot{B}_z-\lim_{\substack{\delta\to 0\\ a\to\infty\\ \sigma\neq 0}}\dot{B}_z,$$

$$-\dot{B}_{x0}\frac{i\sigma\omega}{\delta\beta^2}\cdot\frac{\operatorname{ch}\beta y}{\operatorname{ch}\beta a}=\lim_{\substack{\delta\to 0\\ a\neq\infty}} j_y-\lim_{\substack{\delta\to 0\\ a\to\infty}} j_y;$$

Thus, (39) and (40) may be written as the superposition of the special solutions (which are previously known) and of the special solution in the form of a series, which cannot be reduced to tabulated functions and is not amenable to separation of variables:

$$\begin{gathered}\dot{B}_z=\lim_{\substack{\delta\neq 0\\ a\to\infty\\ \sigma\neq 0}}\dot{B}_z+\lim_{\substack{\delta\to 0\\ a\neq\infty\\ \sigma\neq 0}}\dot{B}_z-\lim_{\substack{\sigma\to 0\\ a\to\infty\\ \sigma\neq 0}}\dot{B}_z+F_B(y,z),\\ j_y=\lim_{\substack{\delta\neq 0\\ a\to\infty}} j_y+\lim_{\substack{\delta\to 0\\ a\neq\infty}} j_y-\lim_{\substack{\delta\to 0\\ a\to\infty}} j_y+F_j(y,z).\end{gathered} \tag{44}$$

(44) is not the only possibility for separating the simpler special solutions from the general solution. Other examples are presented in (Ref. 8, 9). In

this connection, there is a good possibility of employing simpler special solutions of a different type, instead of the total solution, in the appropriate cases. However, for this purpose the magnitude of the discarded portion of the total solution must be determined. A general theoretical analysis of this /84 problem has not yet been conducted. A calculation of the special case (a = 0.13 m, δ = 0.015 m, τ = 0.45 m, liquid potassium at 500°C) on a digital computer has shown that the terms F_b and F_j in (44) can be discarded in this case up to $y = \pm 0.12$ m (in terms of the modulus, they are no less than three orders smaller than the sum of the first three terms) and only in the immediate vicinity of the edge ($y = \pm 0.13$ m) do they become of the same order of magnitude as the remaining terms.

According to (17) and (27), the corresponding attenuation coefficient equals

$$k^{IX}_{oc} = \mathrm{th}^2\,\alpha\delta\,\mathrm{Re}\left\{\frac{i\,\alpha^2 \,\mathrm{cth}\,\beta\delta}{\mu_0\sigma\omega\beta\delta} - \frac{\mathrm{th}\,\beta a}{\beta^3 a\,\delta^2} - \frac{2\alpha^2}{a\delta^2}\sum_{m=1}^{\infty}\frac{\mathrm{th}\,\gamma_m a}{\gamma^3_m\,\gamma'^2_m}\right\}. \tag{45}$$

It may be readily seen that, in the case of $\delta \to 0$, the $k_0{}^{I}$ of A. I. Vol'dek (28) is thus obtained. When verifying this, we must take the fact into account that $\mathrm{th}\,\beta\delta \to \beta\delta$

$$\lim_{\delta\to 0}\frac{\mathrm{th}\,\gamma_m a}{\delta^2\gamma^3_m\,\gamma'^2_m} = 0 \quad \text{and} \quad \mathrm{Re}\frac{i\,\alpha^4}{\mu_0\,\sigma\omega\,\beta^2} = \frac{\alpha^4}{|\beta^4|} = \mathrm{Re}\frac{\alpha^2}{\beta^2}.$$

$k_{oc}{}^{IX}$ is introduced into (45) in exactly the same way as A. I. Vol'dek introduces his $k_{oc}{}^{I}$ (28): a conductive plate having a rectangular cross-section is introduced into the empty clearance between two inductors. The linear current loading of the inductors does not change. Since the clearance between the inductors has finite thickness 2δ, when a change is made from an extremely narrow clearance ($p = p_0\, k_{oc}{}^{IX}$; p_0 is computed for a clearance which is infinitely narrow) to the clearance 2δ, the induction $\dot{B}'_{z0}$ remains constant just as before (the inductors are infinitely wide, and B'_{z0} does not change over their width). Just as previously, we may introduce the attenuation coefficient in the case of a constant supply voltage (in the case of a constant magnetic flux $\dot{\Phi}_\tau$). This yields

$$k^{X}_{oc} = \mathrm{Re}\,\frac{\alpha c^2}{a\delta\mu_0\,\sigma\omega\,\dot{Y}}, \tag{46}$$

where

$$\dot{Y} = i(c-a)\,\mathrm{cth}\,\alpha\delta + \frac{i\alpha a}{\beta}\,\mathrm{cth}\,\beta\delta - \frac{\mu_0\,\sigma\omega}{\alpha\delta\,\beta^3}\,\mathrm{th}\,\beta a - \frac{2\,\mu_0\sigma\omega\alpha}{\delta}\sum_m\frac{\mathrm{th}\,\gamma_m a}{\gamma^3_m\,\gamma'^2_m}; \tag{47}$$

(34) is thus obtained with the limiting transition $\delta \to 0$.

/85

TABLE 1

e	D \ A	0,1	0,3	0,5	1,0	4,0
0,1	0	0,0317	0,2185	0,4151	0,6787	0,9120
	0,02	0316	2180	4140	6769	9096
	0,10	0298	2051	3894	6366	8554
	0,25	0240	1563	2939	4781	6410
	0,50	0158	0917	1638	2577	3403
	1,00	0084	0462	0785	1171	1501
0,5	0	0,0317	0,2150	0,3923	0,5928	0,7483
	0,02	0316	2144	3912	5912	7464
	0,10	0298	2018	3680	5560	7019
	0,25	0240	1539	2780	4181	5269
	0,50	0158	0904	1559	2282	2843
	1,00	0084	0456	0754	1068	1311
1,5	0	0,0316	0,1895	0,2700	0,2951	0,3045
	0,02	0315	1890	2693	2943	3037
	0,10	0297	1778	2534	2769	2857
	0,25	0239	1360	1927	2106	2174
	0,50	0157	0812	1132	1253	1311
	1,00	0084	0416	0581	0667	0721
5,0	0	0,0305	0,0820	0,0649	0,0515	0,0417
	0,02	0305	0818	0647	0513	0416
	0,10	0288	0771	0610	0484	0393
	0,25	0232	0609	0495	0404	0339
	0,50	0153	0408	0366	0331	0306
	1,00	0082	0221	0206	0191	0181
10,0	0	0,0276	0,0305	0,0218	0,0158	0,0114
	0,02	0276	0304	0217	0158	0114
	0,10	0261	0288	0206	0150	0109
	0,25	0211	0245	0187	0148	0119
	0,50	0140	0183	0153	0134	0118
	1,00	0075	0101	0087	0077	0069

Turning to (45), we should point out that k_{oc}^{IX} may be also represented in the following more symmetrical form:

$$k_{oc}^{IX} = \frac{\mathrm{th}^2 \alpha\delta}{\alpha^2\delta^2}\left[(k_{oc}^{I})_0 + 2\sum_{m=1}^{\infty}(k_{oc}^{I})_m\right], \tag{48}$$

where

$$(k_{oc}^{I})_m = \mathrm{Re}\,\frac{\alpha^4}{\gamma_m^2\gamma_m'^2}\left(1-\frac{\mathrm{th}\,\gamma_m a}{\gamma_m a}\right);$$
$$(k_{oc}^{I})_0 = \mathrm{Re}\,\frac{\alpha^4}{\beta^2\alpha^2}\left(1-\frac{\mathrm{th}\,\beta a}{\beta a}\right) = k_{oc}^{I}; \tag{49}$$

$(k_{oc}^{I})0$ is obtained by substituting m = 0 in $(k_{oc}^{I})m$; γ_m^2, $\gamma_m'^2$ are determined by the expressions (41) and (42). /86

The values of k_{oc}^{IX} for several parameters are shown in Table 1. The values of k_{oc} in the case of $D = \frac{\delta}{\tau} = 0$ correspond to the attenuation coefficient of A. I. Vol'dek k_{oc}^{I}. As we may readily see,

$$k_{oc}^{I} = \mathrm{Re}\,\frac{\alpha^2}{\beta^2}\left(1 - \frac{\mathrm{th}\,\beta a}{\beta a}\right)$$

provides a good approximation up to $D = \frac{\delta}{\tau} = 0.02$ (the error does not exceed 0.25%). However, even in the case of $D = 0.10$ only the first sign is valid. The simplified dependence

$$k_{oc}^{III} = 1 - \frac{\mathrm{th}\,\alpha a}{\alpha a}$$

is valid within very narrow limits (in the case of $A = \frac{a}{\tau}$ to 0.1, D -- up to 0.02, and $\varepsilon = \frac{\mu_0 \sigma \omega}{\alpha^2}$ -- up to 1.5). This is clearly inadequate for practical applications, since ε, and particularily A, fall outside of these limits.

We may obtain much better results by a comparatively simple method -- multiplying k_{oc}^{I} by $\frac{\mathrm{th}2\alpha\delta}{\alpha^2\delta^2}$:

$$k_{oc}^{XI} = \frac{\mathrm{th}^2\alpha\delta}{\alpha^2\delta^2}\,\mathrm{Re}\,\frac{\alpha^2}{\beta^2}\left(1 - \frac{\mathrm{th}\,\beta a}{\beta a}\right).$$

In this case, the error does not exceed 1% up to $D = 0.25$ in the case of $\varepsilon \leqslant 1.5$ and $A \geqslant 0.3$, and in the case of $D \leqslant 0.10$ this holds for any A up to $\varepsilon = 10.0$.

Figures 5 and 6 present curves of $\dot{B}_z$, j_y and $\tilde{f}_x$ which were calculated on a computer according to formulas (39) and (40). Figure 5 presents the distribution of these quantities with respect to y in the middle of the channel (in the case of $z = 0$) for the special case mentioned above ($a = 0.13$ m, $\delta = 0.015$ m, $\tau = 0.45$ m. The entire clearance having a thickness of 0.03 m, is filled with potassium at 500°C). Curve 1 gives the amplitude $\dot{B}_z$; 2 - phase; 3 and 4 - the same for j_y; 5 - the mean force density (effective pressure). Figure 6 presents the distribution of these quantities in the case when only part of the clearance is filled with liquid potassium ($a = 0.14$ m, $2\delta = 0.024$ m, $2\Delta = 0.014$ m). In addition, transverse partitions made of stainless steel lower the effective conductivity of the body ($\sigma_{eff} = 0.95 \cdot 10^6$ 1/ohm $\cdot$ m). A re-calculation was performed on the recommendation of A. I. Vol'dek: $\varepsilon \to \varepsilon \frac{\Delta}{\delta}$. /87

It may be seen in Figure 5 that, in the case of a highly conductive medium, the pressure $\tilde{f}_x L$ between the inductors ($\varepsilon = 8.64$) changes greatly over the channel width. It is comparatively simple to combat the decrease in $\tilde{f}_x$ down to zero along the channel edges, since this decrease occurs in a comparatively narrow zone (0.01 m). However, the strongly expressed transverse skin --

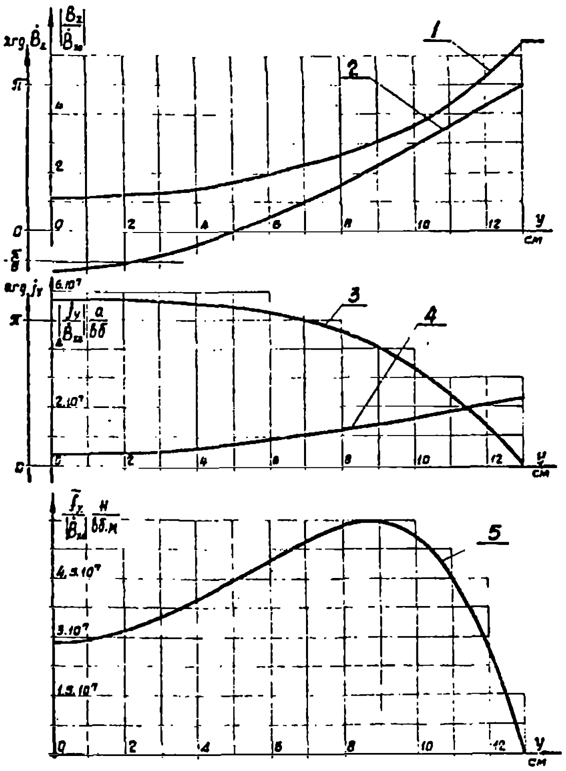

Figure 5

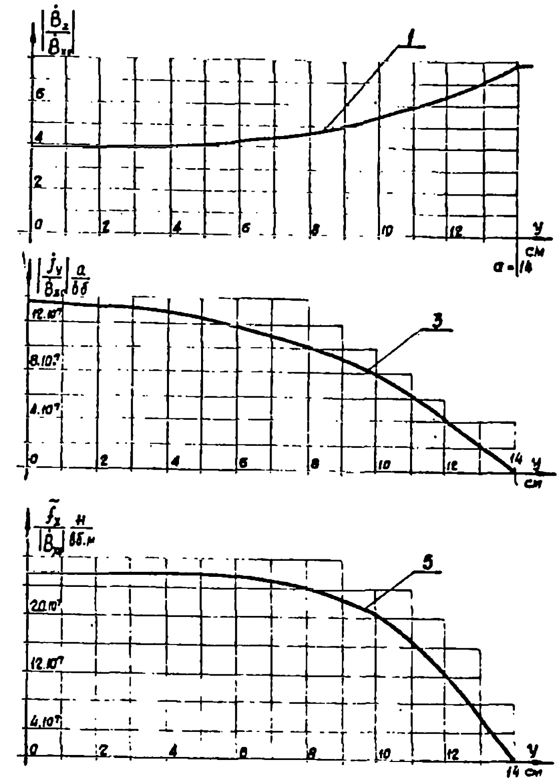

Figure 6

effect -- the decrease in the pressure by almost a factor of two in the wide zone in the center of the channel -- requires special compensatory measures.

The studies of N. M. Okhremenko (Ref. 10, 11) postulate a more complex problem, with the purpose of providing a better approximation of a real, plane pump of finite width: two inductors with a sinusoidal surface current load- /88 ing, which produces a traveling wave; a non-conductive layer of thermal insulation on the surfaces of both inductors; a more conductive, immobile layer (walls of the pump channel) above, below, and in the middle -- a conductive layer /89 which moves like a solid body with a velocity of $v = (1 - s)v_c$ in the direction of displacement of the traveling field wave (Figure 7). The average layer resembles a molten metal in the channel of the pump.

If we check carefully the boundary conditions employed by the author, we shall see that they all apply only on different planes which are perpendicular to the z-axis (parallel to the inductor surfaces). On the flat sides (in the case of $y = \pm a$), no boundary conditions hold. Thus, it would appear that the solution obtained does not depend on the conditions beyond the flat sides of the channel and the inductor. This is an illusion. Based on the general form of the solutions of N. M. Okhremenko, we may readily establish these conditions which hold on the lateral surfaces of the channel $y = \pm a$, and beyond this, and

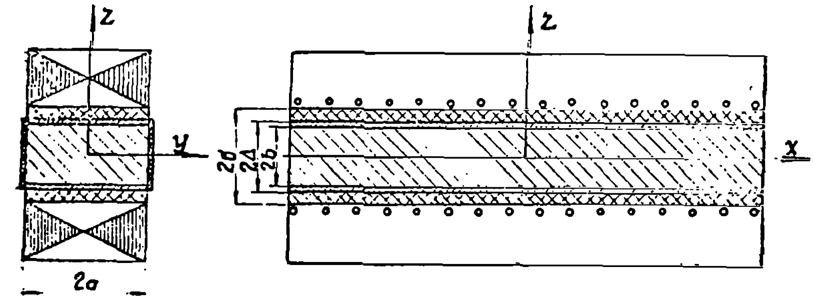

Figure 7

we may formulate the model for which there is a solution of this type.

We would now like to direct our attention to the fact that $\beta_\nu y\big|_{y=\pm a} = 0$, since $\beta_\nu = \frac{2\nu+1}{2a}\pi$ (the notation provided by N. M. Okhremenko). Consequently, in the case of $y = \pm a$ we have $\dot{H}_x = 0$, $\dot{H}_z = 0$, $j_y = 0$ in every region (see sections 5-7, [Ref. 10]). These are the same boundary conditions which would hold if we used surfaces of ferromagnetic media without current loading in the case of $y = \pm a$. Thus, the first possible model which the solution of N. M. Okhremenko describes is as follows. There is a rectangular cavity in a ferromagnetic inductor, in which a single-layered system is located. There is current loading in the form of a traveling wave on the horizontal surfaces of the cavity. There is no current loading on the vertical surfaces. A transverse field component $\dot{H}_y$ must naturally arise in this model.

However, this is not the only possible model. Such a solution is also obtained if the inductors, and also the channel, are infinitely wide, but the current loading over the width of the inductor is distributed according to the following series

$$A_y = A_y \frac{4}{\pi} \sum_{\nu=0}^{\infty} \frac{(-1)^\nu}{2\nu+1} \cos \frac{(2\nu+1)\pi y}{2a}.$$

The sum of this series equals A_y in the case of $-a<y<a$, $3a<y<5a$, $7a<y<9a$, etc., and $-A_y$ in the case of $a<y<3a$, $5a<y<7a$, etc. The current loading on the inductor surfaces changes its sign over the width of the inductors with the period 2a. An infinite succession of inductors arranged side by side is obtained.

The opinion advanced by N.M. Okhremenko, to the effect that the sharp peaks (observed during the experiment) of the component $\dot{B}_y$ along the inductor edges substantiate the validity of the expressions for $\dot{B}_z$ and $\dot{B}_y$ ([Ref. 10], page 23) as well as the similarity between the model under consideration and a real pump, is not convincincing. /90

In the solution of N. M. Okhremenko, a vertical current component ($\dot{E}_z = 0$) is lacking. For this reason, we may make the following statement: the fact that the obvious boundary condition $j_z = 0$ is not employed in the case of $z = \Delta/2$ means simply that it is assumed that $j_z = 0$ over the entire cross-section, as was the case from the very beginning. Otherwise, we would have to set j_z in the form of a series, which yields zeros in the sum in the case of $z = \Delta/2$, but which may yield a non-zero sum for other z. Only if it may be shown that the coefficients of the series j_z equal zero, must we have $j_z = 0$. The presence of $\dot{B}_y$ (due to ferromagnetic lateral surfaces) makes it possible to assume that $j_z = 0$. In this case, there are no apparent contradictions.

Finally, attention must be called to the fact that the boundary conditions (section 2 [Ref. 10])

$$\frac{j_x^{II}}{\sigma^{II}} = \frac{j_x^{III}}{\sigma^{III}s}, \quad \frac{j_y^{II}}{\sigma^{II}} = \frac{j_y^{III}}{\sigma^{III}s}$$

are invalid. Let us try to determine what the boundary conditions must be for the electric field on a plane boundary of two conductive regions which slide over each other and which have complete electrical contact (when changing from one region to another, the current does not encounter any resistance). Region II is immobile (Figure 8); region III slides along it at a velocity of $\mathbf{v} = \mathbf{e}_x \nu = \mathbf{e}_x \nu_n (1 - s) = \mathbf{e}_x \frac{\omega}{\alpha} (1 - s)$. The boundary of the regions passes through the plane $z = b/2$. Let the electric field in region II equal $\mathbf{E}^{II}$ in the case $z = b/2$. The field in region III will be $\mathbf{E}^{*III} = \mathbf{E}^{III} + [\mathbf{vB}]$, where $\mathbf{E}^{III}$ is the field measured in an immobile coordinate system (connected with region II). $\mathbf{E}^{*III}$ is the field in a system which moves together with region III (effective field; $\mathbf{j}^{III} = \sigma^{III}\mathbf{E}^{*III}$). This is the manner in which the Maxwell equations and the data of N. M. Okhremenko are interpreted (section 1 [Ref. 10]).

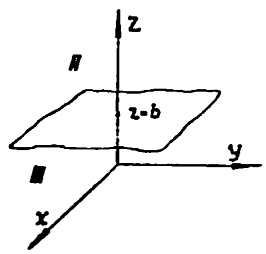

Figure 8

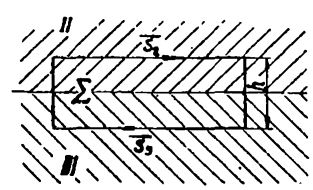

Figure 9

Unfortunately, $\mathbf{B}$ does not undergo a discontinuity on the boundary. /91
We thus have

$$E_x^{*III} = E_x^{III},$$

$$E_y^{*III} = E_y^{III} - v_x B_z,$$

$$E_z^{*III} = E_z^{III} + v_x B_y.$$

The first Maxwell equation is

$$\frac{\partial \mathbf{B}^{II}}{\partial t} = -\operatorname{rot} \mathbf{E}^{II};$$

$$\frac{\partial \mathbf{B}^{III}}{\partial t} = -\operatorname{rot} \mathbf{E}^{III} = -\operatorname{rot} \mathbf{E}^{*III} + \operatorname{rot}[\mathbf{vB}].$$

Let us draw the contour, as is shown in Figure 9, where $h \ll \ell$, $h\ell = \Sigma$, and let us multiply the left and right parts of the equations by a surface element ds. Let us then integrate over every Σ. We apparently have

$$\lim_{h\to 0} \int_{\Sigma} \frac{\partial \mathbf{B}}{\partial t} ds = 0,$$

Since $\mathbf{B}$ is everywhere finite;

$$\lim_{h\to 0} \int_{\Sigma} \operatorname{rot} \mathbf{E}\, ds = \lim_{h\to 0} \oint_{C} \mathbf{E}\, d\mathbf{l} = \int_0^l (\mathbf{E}^{II} - \mathbf{E}^{III})\, d\mathbf{l}.$$

In addition, $\lim\limits_{h\to 0} \int\limits_{\Sigma} \operatorname{rot}[\mathbf{vB}]ds$ depends on the mutual orientation of $\mathbf{v}$ and ds, and equals zero in the case of $\mathbf{v} \perp ds$. However, we do not even require this, since we have the dependence of E^{*III} on E^{III} and $\mathbf{B}$ for the special case. We may then draw the conclusion that, independently of the orientation of $\mathbf{v}$, we have the following on the surface: $E_t^{II} = E_t^{III}$ or, in the given case, in the case of $\mathbf{v} = \mathbf{e}_x v$:

$$E_x^{*III} = E_x^{III} = E_x^{II},$$

$$E_y^{*III} = E_y^{III} - v_x B_z = E_y^{II} - v_x B_z.$$

Changing to currents, we obtain

$$\frac{j_x^{III}}{\sigma^{III}} = \frac{j_x^{II}}{\sigma^{II}}, \quad \frac{j_y^{III}}{\sigma^{III}} = \frac{j_y^{II}}{\sigma^{II}} - v_x B_z \tag{50}$$

instead of section 2 (Ref. 10). The lack of agreement in the first of them /92 is apparent. With respect to the second, we have

$$\dot{B}_z = -\frac{1}{i\omega} \operatorname{rot}_z \dot{\mathbf{E}}^{II} = \frac{1}{i\omega}\left(i\alpha \dot{E}_y^{II} + \frac{\partial \dot{E}_x}{\partial y}\right) = \frac{\alpha}{\omega} \dot{E}_y^{II} + \frac{1}{i\omega} \cdot \frac{\partial \dot{E}_x^{II}}{\partial y};$$

$$\dot{E}_y^{*III} = \dot{E}_y^{II} - \frac{\omega}{\alpha}(1-s)\left(\frac{\alpha}{\omega}\dot{E}_y^{II} + \frac{1}{i\omega} \cdot \frac{\partial \dot{E}_x^{II}}{\partial y}\right) =$$

$$= s\dot{E}_y^{II} + \frac{i(1-s)}{\alpha} \cdot \frac{\partial \dot{E}_x^{II}}{\partial y},$$

And as a result

$$\frac{j_y^{III}}{\sigma^{III}} = s\frac{j_y^{II}}{\sigma^{II}} + \frac{i(1-s)}{\alpha} \cdot \frac{\partial \dot{E}_x^{II}}{\partial y},$$

which coincides with the boundary conditions of N. M. Okhremenko when, and only when,

$$\frac{\partial E_x^{II}}{\partial y} = \frac{\partial E_x^{III}}{\partial y} = 0.$$

This occurs only in infinitely wide channels.

We may draw the following conclusions from the above statements.

At the present time, a sufficiently comprehensive solution has been provided for the problem of the distribution of fields and currents in a rectangular plate between two infinitely wide and long inductors which occupies the entire width of the clearance. The corresponding attenuation coefficients of the pump efficiency are obtained (correction factors to the elementary technical formula for computing the pressure $p_0 = \frac{\sigma\omega L|\dot{B}_{z0}{}^2|}{2\alpha}$). The formulas obtained make it possible to compute the desired quantities readily on a computer. In every case, terms with series of complex hyperbolic functions may be discarded (the sum of the series practically never exceeds the duplicate first term of the series). Then all of the requisite quantities may be expressed in tabulated functions, which may be calculated by hand without any special effort.

The results obtained find limited application in the construction of induction pumps, due to the great idealization of a real pump (solid conductive medium fills the entire clearance, inductors are wider than the medium and are replaced by ferromagnetic half-spaces with sinusoidal surface current loading). These results also find limited application due to the lack of rigorous /93 solutions which provide an estimate of the error involved in this idealization.

One of the problems of paramount importance for future research is the development of a theory for a channel of finite width, whose thickness is less than the clearance between the inductors. In these solutions, the absence of perturbation of the primary field outside of the conductive plate produces a certain effect. This effect must vanish as soon as the plate thickness becomes less than the clearance between the inductors, since the lines of the field will tend to pass around a highly conductive medium. In the limiting case $\varepsilon \to \infty$ complete displacement of the field from the conductive plate must occur. The calculations must be conducted on the basis of the assumption of infinitely wide inductors. The physical meaning of the lack of latteral distortion of the force lines must thus become apparent in the cases under consideration.

The lack of a single theory for an induction pump (pump channel + inductor) reduces the accuracy with which we may calculate such quantities (which are so important in practice) as the active and inductive resistance of the

pump winding and its active and reactive power. It also reduces the accuracy with which we may calculate the changes in these parameters which are caused by changes in the secondary circuit of the pump. This problem is closely related to the problem of errors entailed when a real, tri-phase winding of the inductor is replaced by a system of an infinite number of phases (the higher harmonics are disregarded on the assumption of a sinusoidal form of the current loading).

It is necessary to find a solution for inductors having a finite width, and it is desirable to take into account the influence of frontal sections. Narrow peaks of the transverse induction component have been observed experimentally along the inductor edges. There are theories which either ignore this phenomenon (Ref. 3, 7-9) or else they explain it incorrectly (Ref. 10, 11). The presence of B_y may lead to the re-distribution of E_y and E_x, which quickly has an influence on the pump efficiency.

Finally, it is necessary to find an approximate hydrodynamic solution for a liquid-metal channel which is divided into several parts by longitudnal partitions (this is a case which extremely important in practice). This makes it possible to correctly select structural methods for combating an excessive change in pressure along the channel width and to combat the danger of inverse flows of the molten metal, which greatly reduce the pump efficiency.

The main emphasis must thus be placed on a clear representation of the concepts to be used, without struggling with complex computational formulas. The clear representation of the concepts is absolutely requisite for orienting pump construction in practice (thus, we do not advise an approximate machine solution of the existing differential equations for the given boundary conditions). The computational formulas contain, as a maximum, rapidly-con- /94
verging series of hyperbolic functions of a complex argument. These functions are apparently fundamental solutions in an investigation of induction processes in rectangular regions and may be readily calculated on computers. When one investigates semi-infinite regions (inductor of finite width), it is true that we may expect a solution in the form of Fourier integrals, which may lead to certain difficulties in the numerical calculations.

REFERENCES

1. Shimoni, K. Teoreticheskaya Elektrotekhnika (Theoretical Electrical Engineering). Izdatel'stvo "Mir", 1964.
2. Landau, L.D., Lifshits, Ye.M. Elektrodinamika Sploshnykh Sred (Electrodynamics of Solid Media). Fizmatgiz, 1959.
3. Vol'dek, A.I. Toki i usiliya v sloye zhidkogo metalla ploshikh induktsionnykh nasosov (Currents and Stresses in a Layer of Molten Metal of Flat Induction Pumps). Izvestiya vysshikh uchebnykh zavedeniy, Elektromekhanika, 1, 1959.
4. Vevyurko, I.A. K raschetu kharakteristik dvukhfaznoy induktsionnoy mashiny s polym rotorom (Calculation of Two-Phase Induction Machine with Hollow Rotor). Vestnik Elektropromyshlennosti, 6, 1957.
5. Nitsetskiy, L.V. Modelirovaniye elektricheskogo polya elektromagnitnykh nasosov v elektroliticheskoy vanne i na elektroprovodyashchey bumage

(Modeling of the Electric Field of Electromagnetic Pumps in an Electrolytic Bath and on Electroconductive Paper). V Kn: Elektromagnitnyye protsessy v metallakh (In the Book: Electromagnetic Processes in Metals). Izdatel'stvo AN Latv. SSR, 1959.

6. Ulmanis, L.Ya. Fizicheskiye yavleniya pri induktsionnom vozdeystvii begushchego magnitnogo polya na sloy zhidkogo metalla (Physical Phenomena During the Induction Influence of a Traveling Magnetic Field on a Layer of Molten Metal). Avtoref Kand. Diss. (Author's Abstract of Candidate's Dissertation), Riga, 1963.
7. Vol'dek, A.I., Yanes, Kh.I. Poperechnyy krayevoy effekt v ploskikh induktsionnykh nasosakh pri kanale zhidkogo metalla s provodyashchimi stenkami (Transverse Edge Effect in Plane Induction Pumps in the Case of a Molten Metal Channel with Conductive Walls). V Kn: Voprosy Magnitnoy gidrodinamiki i dinamiki plazmy (In the Book: Problems of Magnetic Hydrodynamics and Plasma Dynamics). Izdatel'stvo AN Latv. SSR, 1962.
8. Vilnitis, A.Ya. Raspredeleniye poley i tokov v provodyashchem tele pryamougol'nogo secheniya, pomeshchennom mezhdy dvumya beskonechnymi induktorami s sinusoidal'nym begushchim magnitnym polem (Distribution of Fields and currents in a Conductive Body Having Rectangular Cross Section, Placed Between Two Infinite Inductors with a Sinusoidal Traveling Magnetic Field). Izvestya AN Latv. SSR, Seriya Fizicheskikh i Teknicheskikh Nauk, 2, 1965.
9. Veske, T.A. Resheniye uraveniy elektromagnitnogo polya ploskoy lineynoy induktsionnoy mashiny s uchetom vtorichnykh poperechnogo i tolshchinogo krayevykh effektov (Soltuion of Equations for an Electromagnetic Field of a Plane, Linear Induction Machine with Allowance for Secondary Transverse Edge Effects and Thick Edge Effects). Magnitnaya Gidrodinamika, 1, 1965.
10. Okhremenko, N.M. Magnitnoye pole ploskogo induktsionnogo nasosa (Magnetic Field of a Plane Induction Pump). Elektrichestvo, 8, 1964.
11. Okhremenko, N.M. Issledovaniye prostranstvennogo raspredeleniya magnitnykh poley i elektromagnitnykh yavleniy v induktsionnykh nasosakh (Study of the Spatial Distribution of Magnetic Fields and Electromagnetic Phenomena in Induction Pumps). Magnitnaya Gidrodinamika, 1, 1965.

LONGITUDINAL EDGE EFFECT IN LINEAR INDUCTION MHD MACHINES /95

Ya. Ya. Valdmanis

1. Introduction

As is well known, the development of a theory for linear induction MHD machines was initiated with research on the simplest idealized models, in which it was assumed that the dimensions of the inductor (primary circuit) and the molten metal (secondary circuit) were infinite in the longitudinal and transverse directions. Refinements related to the finiteness of real MHD machines, which were independent of each other, were introduced in the successive stage. The group of phenomena related to the finiteness of the dimensions in the longitudinal and transverse directions was designated as the longitudinal and

transverse edge effect, respectively. These effects were studied repeatedly both theoretically and experimentally. This article is devoted to an investigation of the present state of research on one of these problems -- the longitudinal edge effect. We would like to point out that a brief summary of this problem may be found in the work by A. I. Vol'dek (Ref. 1).

Before the development of linear MHD machines, the longitudinal edge effect was studied in connection with the development of asynchronous engines with arc and linear stators, but we shall investigate this problem from the viewpoint of MHD machines with allowance for their specific properties (unlimited secondary curcuit and a practically infinite magnetic permeability of the magnetic circuit). Assuming that the channel of molten metal is infinite, from this point on we shall only relate the longitudinal effect to the finiteness of the inductor, which is represented by a smooth magnetic circuit (in theoretical computations) with linear current loading given on its surface, in the form of a traveling wave.

The longitudinal effect of a finite inductor without a secondary circuit is manifested in the presence of additional pulsating fields, apart from the traveling field, in the working clearance. In real inductors of MHD machines, these pulsating fields are propagated over the entire length of the inductor with almost a constant amplitude. We should recall that, in a similar inductor of infinite length in the longitudinal direction, the field in the /96 clearance has the form of an undistorted traveling wave. The formation of pulsating fields in the clearance is an undesirable phenomenon (the symmetry of the currents is distorted, the losses increase, etc.). Therefore, different methods have been advanced for inhibiting this field. When there is a secondary circuit, there is an unusual flow of the induced currents in the molten metal, beyond the limits of the inductor active zone, and the field is carried along in the direction of motion.

Let us first investigate the field of a finite inductor (longitudinal edge effect in a primary circuit), and let us discuss changes in the field distribution when there is a secondary circuit (longitudinal edge effect in a secondary circuit). Finally, we shall point out directions which may be pursued by further research.

Let us first turn to a specific examination of the problem, and we would like to say a few words regarding the general computational method which is applied in almost every study devoted to the longitudinal edge effect.

Only the electrodynamic portion of the computation is investigated in every report -- i.e., the molten metal of the secondary circuit is replaced by a solid metal moving at a constant velocity. All the results are obtained from a solution of the Maxwell equations (in differential or integral forms) with the corresponding boundary conditions.

2. Magnetic Field of a Finite Inductor

The field of a finite inductor was first studied most extensively in a work by G. I. Shturman (Ref. 2), but the possibilities inherent in this study

found practical utilization by A. I.Vol'dek and co-workers (Ref. 3-5, 17).

Let us discuss in greater detail the results derived in these studies. The following problem was studied theoretically in (Ref. 2) (see Figure 1).

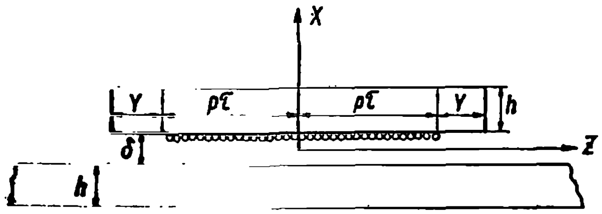

Figure 1

We have a magnetic circuit with the height h and magnetic permeability μ. The current loading is given in the form of $-J_0 \cos(\omega t - \alpha z)$ [in the study (Ref. 2) $J_0 = A$] on the active portion of its length $2p\tau$. In order to /97 make allowance for the shunting currents at the ends of the magnetic circuit, portions of the length Y are left unwound. The lower magnetic circuit, which does not have winding, has the same height h. The air gap equals δ.

Assuming that the magnetic induction in the clearance has only the component B_x (only B_z in the magnetic circuit) and that the magnetic current at the ends of the magnetic circuit (in the case of $z = \pm (Y + p\tau)$ equals zero, we obtain the following for the induction in the clearance in the region of the wound section:

$$B = B_m \sin(\omega t - \alpha z) - (-1)^p B_m [k_c \operatorname{ch} \beta z \sin \omega t - k_s \operatorname{sh} \beta z \cos \omega t], \tag{1}$$

where

$$B_m = \frac{\mu_0 J_0}{\delta(1+\beta^2/\alpha^2)}; \quad k_c = \frac{\operatorname{sh} \beta Y}{\operatorname{sh} \beta (Y + p\tau)}; \quad k_s = \frac{\beta \operatorname{ch} \beta Y}{\alpha \operatorname{ch} \beta (Y + p\tau)}; \tag{2}$$

$$\beta = \sqrt{\frac{2\mu_0}{\mu h \delta}}; \quad \alpha = \frac{\pi}{\tau}. \tag{3}$$

As may be seen from an analysis of expression (1), the first term represents the traveling field, and the last two terms represent, respectively, the portion of the pulsating field which is symmetrical over the inductor length ($\sim \operatorname{ch} \beta z$) and non-symmetrical ($\sim \operatorname{sh} \beta z$). In the case of p which is odd (even), the phase of the symmetrical field coincides (shifted by 180°) with the phase of the traveling field in the middle of the inductor ($z = 0$), and the phase of the non-symmetrical field is shifted up to 90°, as compared with the symmetrical field. If the magnetic permeability of the magnetic circuit strives to infinity [$\beta \to 0$ corresponds to this (3)], the non-symmetrical portion of the field disappears, and the symmetrical component is constant over the inductor length. Consequently, the non-symmetrical component of the pulsating current

is caused by the finiteness μ of the magnetic circuit (saturation). Since the case mentioned above is observed in linear MHD machines, let us write expression (1) in the case of $\beta \to 0$

$$B = B_m [\sin(\omega t - \alpha z) - (-1)^p k_c \sin \omega t], \tag{4}$$

$$k_c = \frac{Y}{Y + p\tau}. \tag{5}$$

For an experimental verification of (4), let us calculate the distribution of the effective induction over the inductor length

$$B_{\text{eff}} = \sqrt{\frac{\omega}{2\pi} \int_0^{2\pi/\omega} B^2 dt} = \frac{B_m}{\sqrt{2}} \sqrt{1 - (-1)^p 2 k_c \cos \alpha z + k_c^2}, \tag{6}$$

from which we readily obtain the maximum value $B_{eff} = B_{max}$, and the minimum value $B_{eff} = B_{min}$, k_c and B_m: /98

$$B_{max} = \frac{B_m}{\sqrt{2}}(1 + k_c); \quad B_{min} = \frac{B_m}{\sqrt{2}}(1 - k_c), \tag{7}$$

$$k_c = \frac{B_{max} - B_{min}}{2}; \quad \frac{B_m}{\sqrt{2}} = \frac{B_{max} + B_{min}}{2}. \tag{8}$$

Figure 2 presents the distribution of the field, which was measured experimentally, in the inductor clearance of one of the pumps constructed at the Institute of Physics of the Latvian SSR Academy of Sciences [similar curves are also given in (Ref. 5, 18)]. The curve corresponding to (6) is shown by the dotted line (this distribution is observed in the case of serrated pulsations).

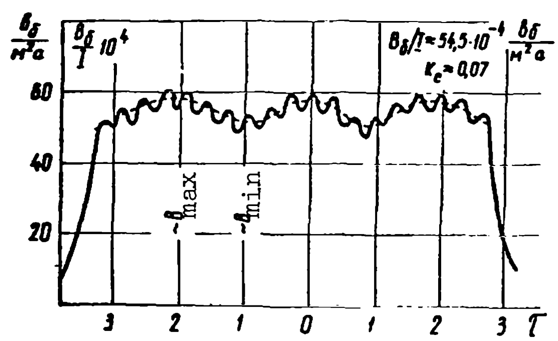

Figure 2

Thus, we may experimentally introduce the coefficient k_c which represents the ratio between the amplitudes of the pulsating and traveling components of the field. However, according to (5) a theoretical introduction of k_c is far from settling this question. It is impossible to determine the quantity **Y**, introduced in (5), theoretically. When we are dealing with real inductors having unwound sections, we may take the length of this section as the provisional

quantity **Y**. The error which is introduced by the currents which are closed at the longitudinal ends of the inductor is thus retained. When there are absolutely no shunting sections at the ends of the inductor, the introduction of the coefficient k_c completely loses any physical meaning, in terms of the calculation indicated above. The study (Ref. 3) proposes that k_c be expressed as follows

$$k_c = \frac{1}{1+\lambda_s}, \tag{9}$$

where λ_s is the ratio between the magnetic conductivity of the shunting /99 sections of the magnetic circuit and the magnetic conductivity of the active zone of the air gap. However, this happens very little in theoretical ratios (the problem may be reduced to calculating currents which are closed at every possible secondary route).

The pulsating component in MHD machines, which is non-symmetrical over the inductor length, is only slightly expressed according to [Ref. 1-3], although this problem not yet been studied sufficiently. The presence of this component may be established experimentally by employing the measurement method developed in (Ref. 21), or by studying the distribution of instantaneous values of the field induction (Ref. 1).

The distribution of the field in a disconnected magnetic circuit was studied by A. A. Lebedev (Ref. 6-9). However, as was already indicated by A. I. Vol'dek (Ref. 1), the results which he obtained do not have practical value due to the errors entailed. We shall discuss below the method which he proposed for balancing the pulsating field.

As was already indicated in the introduction, the formation of supplementary pulsating fields of the edge effect is undesirable, and they must be eliminated as much as possible. Employing the terminology given in (Ref. 3), we shall call this process adjustment of the field, and we shall call the windings with a supplementary pulsating component adjusted windings. Different methods were advanced in (Ref. 3) for balancing the pulsating fields. In the case of a single layered winding, this is achieved by means of a special coil with a current which reaches the core of the stator at the winding level, and which is switched on in phase symmetrically with respect to the inductor core, or with a similar short-circuit loop. In physical terms, this is based on the fact that the field of the coil with the current reaching the core is constant over the inductor length in the case of $\mu = \infty$. We need only select the number of loops of this coil in order that its pulsating field may equal, and be opposite to, the pulsating component of the inductor edge effect. With respect to the short-circuit loop, when there is sufficiently small resistance a current automatically passes through it in such a way that the total induction current through the surface (which it reaches) will equal zero.

For a m-phase single-layered winding, according to (Ref. 3), the number of loops of the compensating coil is

$$w_{\kappa} = \frac{m}{\pi} \cdot \frac{w_0 k_{wi}}{p},$$

where ω_0 is the number of winding loops; k_{wi} is the winding coefficient; p is the number of pairs of poles.

It is thus assumed that the same amount of current passes along the loops of the compensating coil as along the loops of the inductor winding.

For two-layered windings, it is assumed that the half-empty grooves /100
at the inductor ends are filled by the corresponding coils, whose free ends are closed around the edge of the inductor, or are located in special grooves at the ends of the inductor. If the clearance is small, in the region of inverse closure of the compensation windings it is proposed that the clearance be increased and the winding distribution along the length of the section be decreased, in order that the influence of these sections upon the secondary circuit be minimal. The use of a short circuit loop is also possible.

With respect to the compensation method proposed by Blake (Ref. 10), and reiterated by A. A. Lebedev (Ref. 6-9) -- which amounts to a smooth change in the loading to zero at the inductor ends -- it is our opinion that total destruction of the pulsating component may occur. It is true that the question remains open as to how great the drop must be. There is certainly validity in the opinion of A. I. Vol'dek (Ref. 1) that this leads to incomplete utilization of the magnetic circuit, and that this method is barely justified in comparatively short inductors.

The studies (Ref. 1, 3) present arguments regarding the error entailed in this method, and argue that the author (Ref. 10) is only interested in the field in the magnetic circuit yoke. However, as was shown in (Ref. 1, 3), when there is a purely traveling field in the clearance, an additional pulsation current may exist in the yoke. Actually (see Figure 1) in the case of $\beta \to 0$ and $\gamma \to 0$ we may obtain the following from the expression (4) for the magnetic current in the yoke

$$\Phi_{ro} = \int_{-p\tau}^{z} B_m \sin(\omega t - \alpha z)\, dz = \frac{B_m}{\alpha}[\cos(\omega t - \alpha z) - (-1)^p \cos \omega t]. \quad (10)$$

However, the error entailed in the method of Blake does not follow -- i.e., the fact that a pulsation field may exist in the clearance if it is a purely traveling field in the yoke:

$$B_{clear} \sim \frac{\partial B_{ro}}{\partial z}, \quad (11)$$

We thus find that the field in the clearance will be a purely traveling field, if it is a purely traveling field in the yoke.

An article by A. P. Rashchepkin (Ref. 15) recently appeared, in which he

studied windings with linear current loading which was not uniform over the inductor length (in other aspects, the problem was similar to that investigated previously in [Ref. 2], see Figure 1). The loading is represented in the form of a superposition of the windings with different pole divisions (τ) and the same values of the frequencies (ω) and amplitudes of the linear current loadings (J_0). The author (Ref. 15) obtained expressions for the magnetic induction in the clearance in the form of a series, and studied the possibility of balancing the pulsating fields. The method advanced does not differ basically from the method proposed by A. I. Vol'dek which was just discussed. /101

The distribution of the field in the clearance of a finite inductor, as a function of the inductor length, was studied in (Ref. 20). This article investigated the problem, which was similar to that solved by G. I. Shturman (Ref. 2), if we set β and $Y = 0$ in the latter, and if it is assumed that p (length of the inductor in pole divisions) is an arbitrary number. Somewhat, unexpected results were obtained. In this model, the pulsating field was entirely determined by the inductor length. The generalization of expression (4) was also given for the field in the clearance in the region of the wound sections:

$$B=B_m[\cos(\omega t-\alpha z)-\cos\pi p\cos\omega t], \tag{12}$$

where p, as was already indicated, is half of the inductor length in units of the pole division.

In order to suppress the pulsating component, it is proposed that inductors be used whose active portion has an odd number of pole divisions. In this case $\cos\frac{\pi p}{2}=0$ (p -- odd number) and the pulsating portion of the field disappears in (12). In the case of even 2p, we obtain $\cos\pi p = (-1)^p$, which coincides with the results given in (Ref. 3-5, 17). The article (Ref. 20) presents the results derived from experimental studies, which substantiate the validity of (12).

Similar ideas were advanced in (Ref. 10, 19) regarding the possibility of a purely traveling field in the clearance of a finite inductor. However, a sufficiently comprehensive theoretical basis was not provided for these ideas.

3. Field of a Finite Inductor with a Secondary Circuit

All of the preceeding results pertain to no-load operation of the machine. However, the construction of a MHD machine must be optimal in the operational regime. Consequently, it is of greatest interest to study the distribtuion of the field in an inductor with a secondary circuit. If a comprehensive study of this type were carried out, it would be possible to refine the efficiency with which the field could be adjusted during no-load operation. However, as was already indicated in (Ref. 1), this problem has been studied to the least extent, and there is no unified terminology for it. We shall deal with the study performed by A. I. Vol'dek (Ref. 12) in greater detail later on, retaining the terminology employed by him (Figure 3, previous notation). The difference from

the problem investigated above consists of the fact that the magnetic circuit is not limited in terms of height ($h \to \infty$) and in the longitudinal direction /102
($\gamma \to \infty$). Its magnetic permeability if infinite ($\mu \to \infty$) and, in addition, the clearance is filled by a conductor having the conductivity σ which moves like a solid body at the constant velocity $u = u_z$. The study (Ref. 12) also takes into account the presence of a clearance between the inductor and the metal, like serration of the inductor, but this may be reduced to a change in the conductivity of the body which fills the entire clearance. We shall not deal with problem of whether this conductivity of the metal is real or effective, and shall take into account additional phenomena.

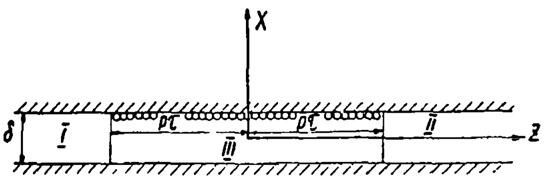

Figure 3

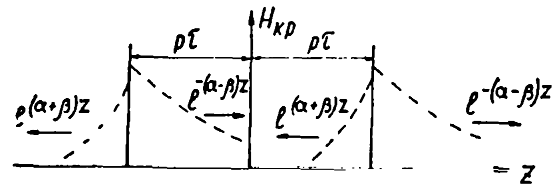

Figure 4

Assuming that the field is a plane-parallel field (i.e., it does not depend on the x-coordinate), for $H = H_x$ we obtain the following equation

$$\frac{\partial^2 H}{\partial z^2} - \mu_0 \sigma \left(\frac{\partial H}{\partial t} + u \frac{\partial H}{\partial z} \right) = 0. \tag{13}$$

The author (Ref. 12) writes the solution within the limits of the active zone as the sum of two fields:

$$H = H_1 + H_2, \tag{14}$$

where

$$H_1 = H_{1\partial} e^{i(\omega t - \alpha z)} + H_{1n} e^{i\omega t}. \tag{15}$$

Thus H_1 is given and differs from zero only within the limits of the active zone ($|z| < p\tau$).

In our opinion, it would be simpler to give the linear loading in the form of a traveling wave -- i.e., only the first term in (15). The pulsating component of the no-load operation [second term in (15)] would have to be obtained from H_2 in (14), when $\sigma \to 0$. This problem is more general as compared with the study by G. I. Shturman (Ref. 2), if $\gamma \to \infty$, $\mu \to \infty$, $h \to \infty$ in the latter, and the results given in (Ref. 2) must be obtained as a special case of the problem under consideration. Such a distinction would be valid if we wished to consider the finiteness of the magnetic circuit or the height of the yoke. However, we would then have to define the pulsating fields beyond the limits of the active zone; these fields exist beyond this zone, as was shown by G. I. Shturman.

For the given case (i.e., $\beta \to 0$ and $\gamma \to \infty$), we obtain /103

$$H_{1n} = -(-1)^p H_{1\delta}. \tag{16}$$

Let us write the formulas derived in (Ref. 12), with allowance for the statements presented above:

$$H_{I} = C_4 e^{(\lambda+\beta)z}, \tag{17}$$

$$H_{II} = C_3 e^{-(\lambda-\beta)z}, \tag{18}$$

$$H_{III} = C_1 e^{-(\lambda-\beta)z} + C_2 e^{(\lambda+\beta)z} - H_{2\delta} e^{-i\alpha z} + H_{1\delta} e^{-i\alpha z}, \tag{19}$$

where

$$H_{2\delta} = \frac{i\varepsilon s H_{1\delta}}{\alpha^2 + i\varepsilon s}; \quad \lambda^2 = \sqrt{\beta^2 + i\varepsilon} = a + ib;$$

$$\beta = \frac{\mu_0 \sigma u}{2}; \quad \varepsilon = \mu_0 \sigma \omega; \tag{20}$$

$$a = \frac{d}{\sqrt{2}}; \quad b = \frac{\varepsilon}{\sqrt{2} d}; \quad d = \sqrt{\beta^2 + \sqrt{\beta^4 + \varepsilon^2}};$$

$$C_1 = \left[\frac{\lambda + \beta + i\alpha}{2\lambda} (-1)^p H_{2\delta} + \frac{\lambda - \beta}{2\lambda} (-1)^{p+1} H_{1\delta}\right] e^{-(\lambda-\beta)p\tau}; \tag{21}$$

$$C_2 = \left[\frac{\lambda - \beta - i\alpha}{2\lambda} (-1)^p H_{2\delta} + \frac{\lambda - \beta}{2\lambda} (-1)^{p+1} H_{1\delta}\right] e^{-(\lambda-\beta)p\tau}; \tag{22}$$

$$C_3 = -2C_2 e^{(\lambda-\beta)p\tau} \operatorname{sh}(\lambda - \beta) p\tau; \tag{23}$$

$$C_4 = -2C_1 e^{(\lambda+\beta)p\tau} \operatorname{sh}(\lambda + \beta) p\tau. \tag{24}$$

The indices I - III designate the total fields in the corresponding regions (see Figure 3).

Let us study in greater detail the structure of the field in the active section ($|z|<p\tau$). We shall only be interested in the additional terms in expression (19), i.e., the first and second terms. The last two terms occur in infinite inductors, and we shall not discuss them.

Thus, with allowance for dependence on time, we may write the field of the edge effect $H_{2\ \mathrm{edge}}$ in the following form

$$H_{2\ \mathrm{edge}} = C_1 e^{-(a-\beta)z} e^{i(\omega t - bz)} + C_2 e^{(a+\beta)z} e^{i(\omega t + bz)}. \tag{25}$$

As may be seen, the fields of the edge effect are fields which move in opposition from the ends of the inductor at a velocity of

$$u_{\mathrm{edge}} = \pm\frac{\omega}{b} = \pm\frac{\sqrt{2} d}{\mu_0 \sigma}. \tag{26}$$

In (Ref. 12), the first field, which is propagated in a positive direction, /104 is called the direct field of the longitudinal effect, and the second field is called the inverse field. As may be seen from (25), in the case of $\beta \neq 0$ (this corresponds to $u \neq 0$), the fields of the edge effect are damped in a dissimilar way: in the case of $u>0$ the direct field is damped more slowly. The field is

carried along by a flux of molten metal, which has been observed experimentally (Ref. 13).

Such a phenomenon has been observed by the author beyond the limits of the active zone. A wave escapes from the right end, which is similar to the wave of the direct edge effect, and a wave which is similar to the wave of the inverse edge effect escapes from the left end. In order to obtain a clearer picture of this, let us examine Figure 4 given in (Ref. 12), which presents a diagram of the distribution and motion of the fields of the edge effect. The arrows designate the direction in which the waves of the edge effect are propagated; their velocity determines (26). Let us present two numerical examples from (Ref. 12): (1) induction pump for aluminum at 735°C; (2) induction pump for sodium at a temperature of 500°C. In both cases, it is assumed that f = 50 cps, the pole division is τ = 0.15 m, and the number of pairs of poles is 2p = 6. The initial data for these examples are presented in Table 1.

TABLE 1

INITIAL DATA FOR THE EXAMPLES

Quantities	Notation	Dimensions	Pump for Aluminum	Pump for Magnesium
Layer Thickness	Δ_0	mm	10	5
Equivalent Clearance	$k_\delta \delta$	mm	35	6.6
Specific Resistance	$1:\sigma$	10^{-8}ohm·m	21.3	18.44
Parameter	ε	$1/m^2$	528	1620

Table 2 presents the quantities for the pumps under consideration which characterize the fields of the edge effect.

TABLE 2 /105

CHARACTERISTICS OF THE FIELDS OF THE EDGE EFFECT

Quantities	Dimensions	Pump for Aluminum			Pump for Sodium		
		s=1	s=0,3	s=0,05	s=1	s=0,3	s=0,05
β	1/м	0	8,85	12,0	0	27,2	36,8
a	1/м	16,2	17,4	18,3	28,4	35,7	41,6
b	1/м	16,2	15,2	14,4	28,4	22,7	19,5
(1) $u_{кр}$	(2) м/сек	19,4	20,6	21,8	11,0	13,8	16,1
$a-\beta$	1/м	16,2	8,55	6,3	28,4	8,5	4,8
$a+\beta$	1/м	16,2	26,2	30,3	28,4	62,9	78,4
$H_{2\partial}:H_{1\partial}$	—	0,772	0,34	0,06	0,965	0,743	0,182

(1) - u_{edge}; (2) - m/sec

As may be seen from the table, the velocity of the edge effect fields increases with a decrease in the slipping and conductivity. Damping of the fields is characterized by $\frac{1}{\beta+a}$ and $\frac{1}{a-\beta}$ (corresponding to the distance at which the inverse and direct fields are damped by a factor of e). Table 2 also presents the ratio between the field of the secondary currents and the field of the primary currents.

It should be pointed out that this phenomenon is nothing else than diffusion of the magnetic field from each end of the inductor in both directions. Passing to the limit $\sigma \to 0$, $u \to 0$ in expression (25) we obtain

$$H_{2\ \text{edge}} = -(-1)^p H_{1z} e^{i\omega t}, \tag{27}$$

i.e., the field of the longitudinal edge effect of the primary circuit. This result is obtained because the field of the direct and inverse longitudinal effect in a secondary circuit is not damped in the limit over the length of the inductor, and its velocity strives to infinity $\sim \frac{1}{\sqrt{\sigma}}$ in the case of $\sigma \to 0$. In regions I and III, the fields disappear, since C_3 and C_4 strive to zero when $u \to 0$ and $\sigma \to 0$.

The longitudinal edge effect in a finite inductor with a secondary circuit was also studied in (Ref. 14) (Figure 5), where -- in contrast to (Ref. 12) -- it is assumed that there is winding on both sides of the channel, the magnetic permeability of the magnetic circuit is μ = const, the inductor length is arbitrary, and in addition damping of the field in the clearance is taken into account (surface effect).

The problem is solved by means of the vector potential A, which is found from solving the equation /106

$$\Delta A - \mu_0 \sigma \left(\frac{\partial A}{\partial t} + u \frac{\partial A}{\partial z} \right) = 0 \tag{28}$$

with the corresponding boundary conditions.

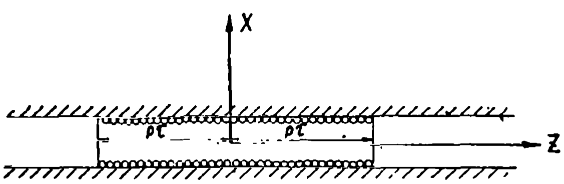

Figure 5

The author (Ref. 14) makes a Fourier transformation with respect to the z variable -- i.e., the solution for A is written in the form

$$A_1 = \frac{1}{2\pi} \int_{-\infty}^{\infty} A_1(\xi, x) e^{i\xi z} d\xi. \tag{29}$$

In view of the symmetry with respect to x, the Fourier component in the clearance may be written as

$$A_I(\xi, x) = A_0 \operatorname{ch}(\sqrt{\xi^2 - iu\mu\sigma + i\omega\mu\sigma})x. \tag{30}$$

However, the assumption is also advanced that in an upper magnetic circuit the

given component of the potential is

$$A_{IV}(\xi, x) = B_0 e^{-\xi x}. \tag{31}$$

It may be readily seen that, by employing the Fourier component (31) we obtain the following for the potential in region IV

$$A_{IV} = \frac{1}{2\pi} \int_{-\infty}^{\infty} B_0 e^{-\xi x} d\xi. \tag{32}$$

Since $x > 0$ in the upper magnetic circuit, this integral diverges, because the integrand increases indefinitely in the case of $\xi \to -\infty$, and consequently A_{IV} (ξ, x) does not have the form of (31).

Since the author (Ref. 14) employs the incorrect Fourier component to obtain all further results, the analysis of this study could be discontinued. The incorrectness of the results, in connection with the statements given above, pertains to all formulas and arguments when μ of the magnetic circuit equals const. However, a large portion of the article(Ref. 14) is devoted to studying the case when μ of the magnetic circuit equals infinity. In this case, the terms, which are obtained from the incorrect form of the potential in region IV, vanish, and a correct result is obtained. The magnetic field in the clearance may be expressed in the form of a series, whose first term coincides with the results derived in (Ref. 12). The author (Ref. 14) has also obtained analytical expressions with allowance for the first term of the series for Joule heat losses for the effective power and consumed power. In addition, the results were analyzed qualitatively for specific values of the parameters introduced, but we shall not present these expressions due to the fact that they are cumbersome. /107

The author (Ref. 14) has also studied the problem of compensation for the longitudinal effect -- i.e., the possibility of obtaining a field in the conductive metal, in the form of a non-distorted traveling wave. In order to do this, it is proposed that winding be employed in the forms of the superposition of three windings with different τ and different amplitudes of the current loadings. However, the use of such complex windings requires an additional theoretical basis or experimental verification.

The field of a finite inductor with a secondary circuit was studied in the work (Ref. 11), but the results obtained (which was indicated in [Ref. 1]) have no practical importance.

The longitudinal effect of a finite inductor without a secondary circuit, and with a secondary circuit, was only studied theoretically in a qualitative manner. To obtain a quantitative theory for the longitudinal effect of a finite conductor, we would have to find more precise methods for determining k_c in expression (4). With respect to the secondary circuit, it would be desirable to clarify the influence of a clearance between the inductor and the molten metal layer upon the decrease in the fields of the edge effect in the secondary circuit (25), since the results given in (Ref. 12, 14) were obtained upon the

assumption that the entire clearance was filled with a molten metal.

It is desirable to provide experimental confirmation of employing inductors with an odd number of pole divisions in the active region, when there is a secondary circuit.

Concurrently with these studies, an analysis must be performed of the longitudinal and transverse edge effects in MHD machines. This problem was studied in (Ref. 16), but -- as was pointed out in (Ref. 1) -- these studies are far from complete. The author (Ref. 16) has provided a very approximate solution of the problem, and only relates the phenomena occurring in the molten metal, within the limits of the inductor active section, with the longitudinal effect.

It would be interesting to study the influence of inductor projections upon the longitudinal effect.

The directions to be followed by future research, which we discussed above, are naturally only provisional, and are far from encompassing all possible paths of research on the longitudinal edge effect in MHD machines.

REFERENCES

/108

1. Vol'dek, A.I. Sostoyaniye i zadachi po razrabotke induktsionnykh nasosov (State of and Problems Involved in the Development of Induction Pumps). Trudy Tallinskogo Politekhn. Instituta, Seriya A, 97, 1962.
2. Shturman, G.I. Induktsionnyye mashiny s razomknutym magnitoprovodom (Induction Machines with a Disconnected Magnetic Circuit). Elektrichestvo, 10, 1946.
3. Vol'dek, A.I. Pul'siruyushchiye sostavlyayushchiye magnitnogo polya induktsionnykh mashin i nasosov s razomknutym magnitoprovodom (Pulsating Components of the Magnetic Field of Induction Machines and Pumps with Disconnected Magnetic Circuit). Nauchnyye Doklady Vysshey Shkoly, Elektromekhanika i Avtomatika, 2, 1959.
4. Vol'dek, A.I. Iskazheniye simmetrii napryazheniy i tokov v induktsionnykh mashinakh i nasosakh s razomknutym magnitoprovodom (Distortion of the Symmetry of Voltages and Currents in Induction Machines and Pumps with Disconnected Magnetic Circuit). Izvestiya Vysshikh Uchebnykh Zavedeniy, Elektromekhanika, 5, 1960.
5. Vol'dek, A.I., Vyal'yamyae, G.Kh., Sillamaa, Kh.V., Tiysmus, Kh.A. Eksperimental'nyye essledovaniya magnitnykh poley v induktsionnykh mashinakh i nasosakh dlya zhidkikh metallov s razomknutym magnitoprovodom (Experimental Studies of Magnetic Fields in Induction Machines and Pumps for Liquid Metals with Disconnected Magnetic Circuit). Trudy Tallinskogo Politekhn. Instituta, Seriya A, 131, 1958.
6. Lebedev, A.A. Gidrodvigatel' so sfericheskim massivnym rotorom (Hydraulic Engine with a Massive, Spherical Rotor). LKVVIA, 1959.
7. Lebedev, A.A. Umen'sheniye poter' ot krayevogo effekta dugovogo statora (Decrease in Losses from the Edge Effect of an Arc Stator). Trudy

LKVVIA, 282, 1959.
8. Lebedev, A.A. Magnitnoye pole v zazore asinkhronnoy mashiny s dugovym statorom, vozbuzhdayemoye pervichnoy obmotkoy (Magnetic Field in the Clearance of an Asynchronous Machine with an Arc Stator, Excited by Primary Winding). Izvestiya Vysshikh Uchebnykh Zavedeniy, Elektromekhanika, 5, 1959.
9. Lebedev, A.A. Magnitnoye pole, vozbuzhdayemoye dugovym statorom (Magnetic Field Excited by an Arc Stator). Trudy LKVVIA, 295, 1959.
10. Blake, R.L. Conduction and Induction Pumps for Liquid Metals. Proc. IEE, 104, part A, 13, 1957.
11. Shturman, G.I., Aronov, R.P. Krayevoy effekt vo vtorichnoy tsepi induktsionnykh mashin s razomknutym magnitoprovodom (Edge Effect in the Secondary Circuit of Induction Machines with Disconnected Magnetic Circuit). Elektrochestvo, 2, 1947.
12. Vol'dek, A.I. Prodol'nyy krayevoy effekt vo vtorichnoy tsepi induktsionnykh mashin i nasosov dlya zhidkikh metallov s razomknutym magnitoprovodom (Longitudinal Edge Effect in the Secondary Circuit of Induction Machines and Pumps for Liquid Metals with Disconnected Magnetic Circuit). Izvestiya Vysshikh Uchebnykh Zavedeniy, Elektromekhanika, 3, 1960.
13. Rezin, M.G. Effekt reaktsii yakorya i mekhanicheskiye kharakteristiki dvigatelya s dugovym statorom (Effect of Armature Reaction and Mechanical Characteristics of an Engine with an Arc Stator). Elektrichestvo, 2, 1950.
14. Sudan, R.N. Interaction of a Conducting Fluid Stream with a Traveling Wave of Magnetic Field of Finite Extension. J. Appl. Phys., 34, 3, 1963.
15. Rashchepkin, A.P. Pole v zazore pri peremennoy lineynoy nagruzke obmotki induktsionnoy mashiny (Field in the Clearance in the Case of Variable Linear Loading of an Induction Machine Winding). Magnitnaya Gidrodinamika, 3, 1965.
16. Okhremenko, N.M. Elektromagnitnyye yavleniya v ploskikh induktsionnykh nasosakh dlya zhidkikh metallov (Electromagnetic Phenomena in Plane Induction Pumps for Liquid Metals). Elektrichestvo, 3, 1960.
17. Vol'dek, A.I. Kompensatsiya pul'siruyushchego magnitnogo polya v asinkhronnykh mashinakh i induktsionnykh nasosakh s razomknutym magnitoprovodom (Compensation for Pulsed Magnetic Field in Asynchronous Machines and Induction Pumps with Disconnected Mangetic Circuit). Elektrichestvo, 4, 1965.
18. Ulmanis, L.Ya. K voprosu o krayevykh effektakh v lineynykh induktrionnykh nasosakh (Problem of Edge Effects in Linear Induction Pumps). Trudy Instituta Fiziki AN Latv. SSR, 8, 1956.
19. Watt, D.A. A Study in Design of Traveling Field Electromagnetic Pumps for Liquid Metals. Harwel 1955.
20. Valdmanis, Ya.Ya., Liyelpeter, Ya.Ya. K teorii prodol'nogo krayevogo effekta lineynoy induktsionnoy magnitogidrodinamicheskoy mashiny (Theory of Longitudinal Edge Effect of a Linear, Induction, Magnetohydrodynamic Machine). In Press.
21. Ulmanis, L.Ya. Izmereniye sostavlyayushchikh induktsiy magnitnogo polya v zazore induktsionnogo nasosa (Measurement of Components of Magnetic Field Induction in Clearance of an Induction Pump). V Kn: Voprosy magnitnoy gidrodinamiki (In the Book: Problems of Magnetic Hydrodynamics),3. Izdatel'stvo AN Latv. SSR, 1963.

PONDEROMOTIVE FORCES ACTING UPON CONDUCTIVE BODIES IN THE TRAVELING MAGNETIC FIELD OF A CYLINDRICAL INDUCTOR

Yu. K. Krumin'

1. Introduction

Up to the present time, the traveling magnetic field with axial symmetry has been studied less frequently than fields of the flat type. However, it is of great importance for the development of new designs for MHD machines. The advantage of fields with axial symmetry lies in the fact that inductors for producing such fields have no frontal sections, and there are no transverse edge effects in the working medium. This leads to an increase in the efficiency of the machine, as a whole.

However, cylindrical inductors in electromagnetic pumps and other MHD machines have been employed to a lesser extent than flat inductors. There are few articles in the literature on this problem (Ref. 19, 22). The use of a traveling magnetic field of cylindrical inductors is also well known for producing a different type of electromagnetic conveyers of solid conductive objects (Ref. 6, 7). The reason for this may be found primarily in the fact that the distribution of the force density over the channel cross-section is different in a cylindrical pump (with a solid channel, without an inner core) than it is in a flat pump: it equals zero in the center of the channel. Due to this fact, a different type of closed flows may occur, which obstruct the normal operation of the pump. One opinion even holds that a cylindrical pump without a core cannot operate successfully, in general. Only recently have studies appeared (Ref. 2, 17), in which it was shown that this opinion is incorrect, and that the construction of cylindrical pumps without cores is fully possible.

A ferromagnetic core considerably improves the situation, and therefore it is employed in all existing cylindrical pumps. However, it is only employed under conditions in which the core still retains its ferromagnetic properties.

One of the main problems which is encountered when designing any device utilizing ponderomotive forces, which influence the conductive media located in the traveling magnetic field, is the development of methods for calculating these forces, particularly their maximum values, as a function of other /110 characteristics of the device. This chapter will be devoted to these problems.

2. Cylindrical Inductor and its Electromagnetic Field

As is known, the cylindrical inductor of a traveling magnetic field represents a system of coils located on a common axis, which are supplied by an alternating current which is usually a tri-phase current. The field produced by such an inductor is rather complex. Therefore, in theoretical designs simplifications are usually introduced which assume that the conductive medium

under consideration is located in the idealized field produced by an ideal inductor, and not in the field of a real inductor. A cylindrical surface having the radius R of infinite length, around whose circumference a surface current flows, is placed under an ideal inductor. Its density changes sinusoidally both in time and along the z-coordinate: $A = A_0 e^{i(\omega t - \alpha z)}$. In other words, this current represents a traveling wave, whose phase propagation velocity equals ω/α. At such a velocity, the equal phase plane of the field excited by this current is displaced along the z-coordinate.

The electromagnetic field in this inductor -- i.e., in the case of $r \leqslant R$ -- has the following form if the inductor is filled with a uniform medium having the specific conductivity σ and the magnetic permeability μ:

$$\left\{\begin{aligned} H_r &= \frac{i\alpha H_{00}}{\beta} \cdot \frac{I_1(\beta r)}{I_0(\beta R)} e^{i(\omega t - \alpha z)}; \\ H_z &= H_{00} \frac{I_0(\beta r)}{I_0(\beta R)} e^{i(\omega t - \alpha z)}; \\ E_\varphi &= -\frac{i\omega\mu H_{00}}{\beta} \cdot \frac{I_1(\beta r)}{I_0(\beta R)} e^{i(\omega t - \alpha z)}. \end{aligned}\right. \tag{1a}$$

Here we have

$$\beta = \sqrt{\alpha^2 + i\sigma\mu\omega}, \tag{2}$$

and H_{00} is the strength of the tangential (H_z) component of the magnetic field on the inductor surface -- i.e., in the case of $r = R$.

If there are no conductive and ferromagnetic media within the inductor, the field has the following form /111

$$\left\{\begin{aligned} H_r &= iH_{00} \frac{I_1(\alpha r)}{I_0(\alpha R)} e^{i(\omega t - \alpha z)}; \\ H_z &= H_{00} \frac{I_0(\alpha r)}{I_0(\alpha R)} e^{i(\omega t - \alpha z)}; \\ E_\varphi &= -\frac{i\omega\mu_0 H_{00}}{\alpha} \cdot \frac{I_1(\alpha r)}{I_0(\alpha R)} e^{i(\omega t - \alpha z)}. \end{aligned}\right. \tag{1b}$$

Figure 1 presents a picture of the magnetic field of an empty inductor for two cases, when the ratio $2R/\tau$ equals 2 and 0.5.

The electromagnetic field of a real inductor may be computed by summing up the fields of all the loops along which the current passes. Such a calculation was performed in (Ref. 16), in which an expression was found for the radial (H_r) component of the magnetic field for an inductor of finite length. /112 It was also assumed that the inductor winding was infinitely narrow, and the inductor itself represented a group of solenoids located on a common axis and arranged close to each other.

Since the magnetic field of a solenoid of finite length is well known (Ref. 14), no particular difficulties are entailed in determining the total field of

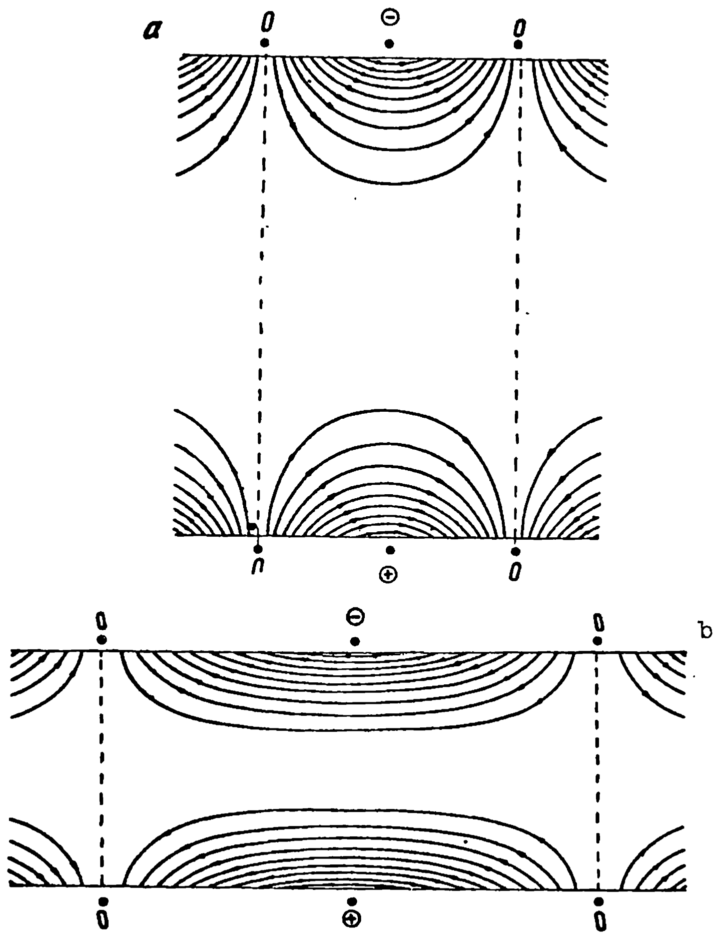

Figure 1

Form of the Magnetic Field in a Cylindrical Inductor:

a - For $2R/\tau = 2$; b - For $2R/\tau = 0.5$

the inductor. Employing dimensionless notation, we may represent the radial component of the field of such an inductor in the following form

$$\overline{H}_r = \frac{2\pi H_r}{\mu A_0} = \sum_{n=1}^{k} \sqrt{\frac{1}{r}} \left\{ \left[\left(\frac{2}{k_1} - k_1 \right) K(k_1) - \frac{2}{k_1} E(k_1) \right] - \right.$$
$$\left. - \left[\left(\frac{2}{k_2} - k_2 \right) K(k_2) - \frac{2}{k_2} E(k_2) \right] \right\} \cos \left[\omega t + \frac{(n-1)\pi}{3} \right]. \tag{3}$$

Here E and K are the complete elliptic integrals of the first and second type;

$$k_1=\sqrt{\frac{4\bar{r}}{(1+\bar{r})^2+\left(\frac{n\bar{\tau}}{3}-\bar{z}\right)^2}};\quad k_2=\sqrt{\frac{4\bar{r}}{(1+\bar{r})^2+\left[\frac{(n-1)\bar{\tau}}{3}-\bar{z}\right]^2}};\tag{4}$$

k is the number of solenoids forming the inductor;

$$\bar{r}=\frac{r}{R};\ \bar{\tau}=\frac{\tau}{R};\ \bar{z}=\frac{z}{R}.\tag{5}$$

The values of H_z and E_ϕ are not given in the work (Ref. 16).

Figure 2 presents a graphic illustration of the function (3) for the moment of time t = 0 at a value of $\bar{r}$ = 0.5 for certain $\bar{\tau}$.

The field of a real inductor differs from the field of an ideal inductor (if we do not assume that it is distorted close to the ends) in the fact that in a real inductor it is not a sinusoidal field along the direction in which it is propagated -- z-axis. It may be expanded in series of spatial harmonics, which all change at the same rate over a period of time, but which have different τ. The larger the ratio of the pole step to the inductor radius, the larger is the amplitude of these harmonics. This is clearly illustrated in Figure 2. In the case of small $\bar{\tau}$, the form of the curve is close to a sinusoid, but its amplitude is small. The conclusion may thus be reached that a certain optimum τ_{optim} must exist, at which the operation of the electromagnetic device will be the most efficient.

The distribution of the magnetic field (averaged over time) in the real cylindrical inductors along the inductor length was studied in (Ref. 3). The influence of individual coils and projections of the inductor magnetic circuit upon the field may be clearly seen in the graphs. /113

In addition to studies which calculate the electromagnetic field in an inductor filled with a unifrom substance, there are still many solutions of the problem when a conductive medium having a certain configuration is placed in the inductor. We shall investigate these solutions, together with the corresponding solutions for the ponderomotive forces, since they represent two sides of the same problem.

3. General Formulation of the Problem and Method of Solution

As has already been indicated, the problem of a cylinder having infinite length, located in the field of an ideal inductor, has been subjected to the most extensive theoretical treatment. Although this represents an idealization of reality, the results obtained nevertheless present a correct concept of the phenomenon -- the nature of the dependence of force on frequency, conductivity, pole division, etc. The contribution furnished by a consideration of the real conditions amounts to a correction of the solution obtained. In addition, it may be stated that -- if we know the spatial harmonics of the field -- then, in view of the superposition principle, we may obtain the complete solution by summing up all of the special solutions which correspond to each harmonics. All of this

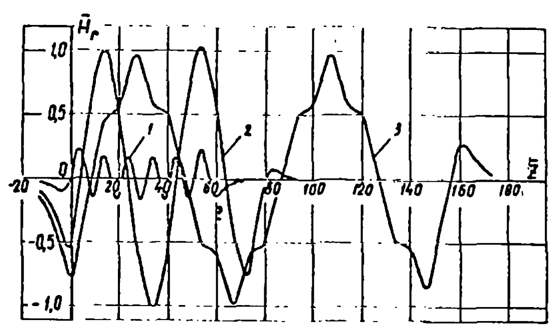

Figure 2

Dependence of $\bar{H}_r$ for a Real Inductor Having Finite Length Upon $\bar{z}$ in the Case of $r = 0.5$ and Certain Values of $\bar{\tau}$: 1 - $\bar{\tau}=0.5$; 2 - $\bar{\tau}=2.0$; 3 - $\bar{\tau}=4.0$.

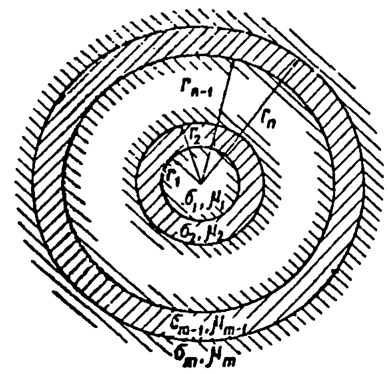

Figure 3

Cross-Section of Coaxial System When the General Case if Calculated.

speaks in favor of a detailed study of the idealized case, which may thus be assumed to be fundamental for a study of real cases.

The solutions for ideal cases, which are presented in different studies, differ primarily in the amount and order in which the layers are arranged (layers of coaxial cylinders), which are located in the inductor. For all of these problems, we may present a general method which may be employed to derive the solution, which is as follows. /114

Let us assume the most general case, when the device consists of n coaxial cylinders, along which circular surface currents pass (we shall assume that they have the same ω and α). These currents alternate with m uniform layers, whose specific conductivity and magnetic permeability equals σ_k and μ_k ($k = 1, 2, \ldots, m$) (Figure 3). The radii of the surface dividing the media may be designated by r_k.

Let us solve the Maxwell equations in cylindrical coordinates. Since the phenomenon does not depend on the angle ϕ in this case, all of the derivatives with respect to ϕ in these equations must be set equal to zero. Due to this fact, the system of equations is broken down into two independent systems (each one consists of three equations with three unknowns). One of them contains H_r, H_z and E_ϕ, and the other contains E_r E_z and H_ϕ. It may be readily seen that $E_r = E_z = H_\phi = 0$, and the solution of the first system has the following form

$$\begin{cases} H_{rk} = -\frac{\alpha}{\omega\mu_k}[C_{1k}I_1(\beta_k r) + C_{2k}K_1(\beta_k r)]\, e^{i(\omega t - \alpha z)}; \\ H_{zk} = \frac{i\beta_k}{\omega\mu_k}[C_{1k}I_0(\beta_k r) - C_{2k}K_0(\beta_k r)]\, e^{i(\omega t - \alpha z)}; \\ E_{\varphi k} = [C_{1k}I_1(\beta_k r) + C_{2k}K_1(\beta_k r)]\, e^{i(\omega t - \alpha z)}, \end{cases} \tag{6}$$

where

$$\beta_k = \sqrt{\alpha^2 + i\sigma_k\mu_k\omega} \qquad (k = 1, 2, \ldots, m). \tag{7}$$

Such a solution must be written for all m layers, substituting the corresponding values of σ_k and μ_k, with the exception of the extreme values -- of the inner layer containing points with r = 0 and the outer layer containing points with r = ∞. In order that the solutions may be finite and unique, we must discard the functions K in the inner layer, and the functions I in the outer layer.

The integration constants C_{1k} and C_{2k} are determined from the boundary /115 conditions, which hold on each dividing surface, and which have the following form

$$\left.\begin{aligned} \mu_k H_{rk} &= \mu_{k+1} H_{r,\,k+1} \\ H_{zk} - H_{z,\,k+1} &= A_k \\ E_{\varphi k} &= E_{\varphi,\,k+1} \end{aligned}\right\} \text{ for } \quad r = r_k. \tag{8}$$

It may be readily seen that the first and third conditions of (8) yield identical equations, and we thus obtain a system of equations from 2k - 2 equations with 2k - 2 unknowns. With respect to the second condition of (8), if no surface current flows along the dividing surface, then zero must be substituted instead of A_k.

The solution obtained completely determines all of the electromagnetic processes in the system under consideration. However, the complexity of the computations increases greatly with an increase in the number of layers, and therefore only problems with several layers have been investigated more or less extensively at the present time.

Cylindrical pumps with the number of layers reaching 6 may be frequently encountered in practice (Ref. 1). In this connection, E. K. Yankop (Ref. 20) proposed a method for solving this system of equations in the case of a large number of equations.

4. Methods of Representing the Results

The direct purpose of the computation is to determine the ponderomotive forces influencing the working layer in the inductor.

In this case, there are two force components which differ from zero -- the radial component f_r which contracts the cylinder, and the tangential component

f_z which operates in the direction of motion of the field. Their density averaged over time may be computed according to the well known formulas:

$$\begin{cases} \bar{f}_r = \frac{1}{2}\operatorname{Re}[j_{\varphi 0} B_{z0}^*] = \frac{\sigma}{2}\operatorname{Re}[E_{\varphi 0} B_{z0}^*]; \\ \bar{f}_z = -\frac{1}{2}\operatorname{Re}[j_{\varphi 0} B_{r0}^*] = -\frac{\sigma}{2}\operatorname{Re}[E_{\varphi 0} B_{z0}^*]. \end{cases} \quad (9)$$

The total force may be found by integrating the force density over the entire volume of the conductive substance. Thus, the radial forces are mutually balanced, and therefore produce no effects if the cylinder under consideration is solid. When they influence a liquid, different radial flows may arise.

The force component operating in the direction of motion of the field produces the useful effect of displacing the conductive body. Therefore, /116
when one speaks of the forces in the traveling magnetic field, it is usually this component which one has in mind, unless otherwise stipulated.

We must now turn to the method of defining the boundary conditions. It must be stated that (8) is not the only possible method for defining them. Instead of defining the current values in the inductor, we may define the strength of the magnetic field on any dividing surface. This is the customary procedure, especially when there is only one surface along which the surface currents pass. Thus, the strength (H_z, H_r) is given, both on the surface where the currents pass, and on the surface of the conductive cylinder which is located in the inductor. In other words, one and the same problem may be represented in a completely different manner, and the expressions for the force will also be different. When we are computing the force, we are only interested in the processes in the conductive medium under consideration. Therefore, we shall assume that it is more advantageous to divide the problem into two parts: to define the strength of the magnetic field (one of its components) on the surface of the conductive medium (if it is solid), with which the electromagnetic processes within it will be clearly determined, and to calculate this strength by the method indicated above as a function of the currents flowing in the inductor windings, as well as other characteristics of the device. It is possible to define the strength of the field on the surface of the medium, because the distribution of the electromagnetic fields does not change as a function of the surface on which we define the boundary conditions.

If the medium which we are considering is a hollow body, it will be insufficient to define the magnetic field only in the form of a single boundary condition on its durface. We must here define two conditions. This may be done by defining the values of both field components from one side of the body, or with respect to one component from the outer and inner side. However, such a method is not employed in practice. By employing this method, for a specific form of the body we shall only have one formula for the force; we shall have to substitute a specific value of the magnetic field strength in each case in this formula. This makes it possible to compile the general curves or tables for the calculations, which may be employed in every case, and to analyze the phenomenon, abstracting from the foreign influences introduced by other elements of the device.

Naturally, all of these statements only pertain to problems regarding infinitely long cylinders in an ideal inductor.

In order to decrease the number of variables, the computational results may be represented in dimensionless form. Thus, the following dimensionless numbers are introduced as the arguments: /117

$$\bar{\omega}=\sigma\mu\omega a^2; \quad \bar{\alpha}=\alpha a, \tag{10}$$

where a is a certain characteristic dimension. Customarily, if a solid cylinder is being considered, its radius is taken as a. For an empty cylinder, it is natural to use its wall thickness as a. In several articles $\bar{\omega}$ is called the relative frequency.

In addition, it is advantageous to intorduce one dimensionless condition -- the complex relative frequency $\bar{\omega}_i$:

$$\bar{\omega}_i=\beta a=\sqrt{\frac{\sqrt{\bar{\omega}^2+\bar{\alpha}^4}+\bar{\alpha}^2}{2}}+i\sqrt{\frac{\sqrt{\bar{\omega}^2+\bar{\alpha}^4}-\bar{\alpha}^2}{2}}=\gamma_1+i\gamma_2. \tag{11}$$

The quantities $\bar{\omega}$ and $\bar{\omega}_i$ have a simple physical meaning: they indicate that a variable field penetrates to a portion of the characteristic dimension a. Thus, $\bar{\omega}$ characterizes the penetration of a variable, non-traveling field, and the real portion $\bar{\omega}_i$ characterizes the traveling magnetic field (Ref. 8):

$$\frac{h_1}{a}=\frac{1}{\sqrt{\bar{\omega}}}; \quad \frac{h_2}{a}=\frac{1}{\gamma_1}, \tag{12}$$

where h_1 and h_2 represent the effective penetration depth, respectively, of the variable non-traveling field and the traveling field in half-space with a plane boundary dividing the media.

As regards representation of the force in dimensionless form, the situation is more complex, since different variants are possible. In this connection, different determinations of the dimensionless force are encountered in different articles.

We shall assume that we have calculated the total force on a segment of a cylinder having the length ℓ. Intorducing the dimensionless parameters $\bar{\omega}$ and $\bar{\alpha}$ which have already been indicated, we find that this force may be represented as follows

$$F\sim\frac{l\mu H_0^2}{\alpha}f(\bar{\omega},\bar{\alpha}). \tag{13}$$

If we divide both sides of this equation by the factor in front of $f(\bar{\omega},\bar{\alpha})$ and if we designate the left side by $\bar{F}$ (as, for example, was done in [Ref. 12]), we then find that $\bar{F}$ depends on α -- a quantity on which $\bar{\alpha}$ also depends (in addition, $\bar{\omega}$ and $\bar{\alpha}$ depend on a). This renders an interpretation of the results more difficult. /118

We may select a dimensionless condition $\bar{F}$ which does not include any of the quantities contained in $\bar{\omega}$ and $\bar{\alpha}$. This condition is as follows*:

$$\bar{F}=\frac{F_{cr}}{\mu H_0^2}. \tag{14}$$

However, F_{cr} does not have the dimension of force here, although in physical terms it may be regarded as a force acting upon a certain volume of substance (this will be demonstrated in specific problems). F_{cr} has the dimension of pressure, but at the same time it is not pressure which may be produced by an apparatus.

In this section, we shall adhere to the opinion that it is better to proceed with a certain complication of the physical meaning of $\bar{F}$, than it is to apply interdependent conditions, whose relationship is difficult to interpret in physical terms.

The separation of the problem into two stages, as was mentioned previously, is also advantageous when one wishes to present a graphic illustration of the results. The relationship between no more than three quantities may be depicted in the form of a set of curves on a plane, but the solution of the general problem, even in the simplest cases, usually depends on no less than four quantities. Nomography would be of considerable help, but nomograms have not as yet been compiled for the results investigated in our article. A graphic illustration is completely impossible, since all the formulas are very complex for computations by hand.

We would like to point out the following. If we use ω to designate the angular frequency of a current which is supplied to an inductor, all the formulas of this article reflect the processes occurring when the conductive media are not in motion. When the conductive medium moves with respect to the inductor, all the formulas remain in force, but -- instead of ω -- we must substitute another value of the frequency ω_1 in these formulas. This value depends on the slipping S:

$$\omega_1=\omega S. \tag{15}$$

In the case of $S = 0$, when the medium and the traveling field move at the same velocity, all the forces are equal to zero. In the case of $S < 0$, these formulas will also be real, but the value of the forces will have to be assigned the opposite (negative) sign.

5. Solid Conductive Cylinder in the Field of an Ideal Inductor /119

An attempt was made to solve this problem in a report by A. I. Tyutin (Ref. 18). However, he only calculated the force density, and did not integrate it over the cylinder volume. This problem was studied in greater detail in

* Strictly speaking, $\bar{F}$ and $\bar{\omega}$ depend on μ. However, since we usually only deal with substances in which $\mu = \mu_0$, in this context they are independent.

(Ref. 12, 13), where the authors calculated the force acting upon a solid conductive cylinder with the radius ρ, located in the traveling magnetic field of a coaxial inductor. If we assume that the z-component (having the amplitude H_0) of the magnetic field strength is given on the cylinder surface, we obtain the following result

$$\bar{F}=\frac{F_{cr}}{\mu H_0^2}=\frac{2\bar{\alpha}\,\mathrm{Im}\,[\bar{\omega}_i I_0(\bar{\omega}_i) I_1(\bar{\omega}_i^*)]}{|\bar{\omega}_i I_0(\bar{\omega}_i)|^2}. \tag{16}$$

We shall employ F_{cr} to designate the force acting upon a volume of substance of 2ρ units of volume. Figure 4 presents a graphic illustration of dependence (16), where values of the condition $2\rho/\tau = 2\bar{\alpha}/\pi$ which are more suitable for practice are given, instead of $\bar{\alpha}$. This dependence represents curves which have one maximum.

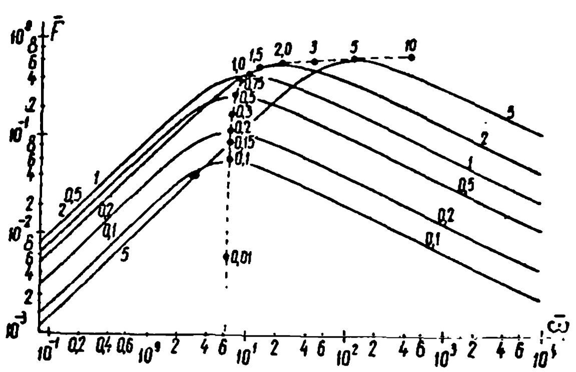

Figure 4

Dependence of $\bar{F}$ on $\bar{\omega}$ for Different $2\rho/\tau$ for a Solid Cylinder

In the case of $\bar{\omega} \ll \bar{\alpha}$, the force increases in direct proportion to $\bar{\omega}$, according to the approximate formula

$$\bar{F}\approx\frac{\{I_1(\bar{\alpha})[I_0(\bar{\alpha})+\bar{\alpha}I_1(\bar{\alpha})]-I_0(\bar{\alpha})[\bar{\alpha}I_0(\bar{\alpha})-I_1(\bar{\alpha})]\}\,\bar{\omega}}{\bar{\alpha}^2 I_0^2(\bar{\alpha})}, \tag{17}$$

and in the case of $\bar{\omega} \gg \bar{\alpha}$ it strives to zero in inverse proportion to $\sqrt{\bar{\omega}}$:

$$\bar{F}\approx\frac{\sqrt{2\bar{\alpha}}}{\sqrt{\bar{\omega}}}. \tag{18}$$

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The maxima of these curves are of special interest. However, since we still do not know how to derive the expressions for them analytically, we shall confine ourselves to analyzing their behavior in a figure. With an increase in the ratio between the cylinder diameter 2ρ and the pole step of the inductor τ, the maxima $\bar{F}$ strive to the largest possible value, equalling approximately 0.85. For small ratios of $2\rho/\tau$, all the maxima occur for one specific value of $\bar{\omega}$, equalling 6.33. For small values of $\bar{\omega}$, the maximum force is reached at one specific value of $2\rho/\tau$, equalling 0.92 (in the case of $\bar{\omega} \lesssim 15$; $2\rho/\tau \lesssim 1.5$).

The study (Ref. 13) presents formulas and curves for calculating H_0 with respect to the inductor characteristics for the following case: the diameter of the inductor is R; there is a non-magnetic, non-conductive clearance between the conductive cylinder and the inductor; the space outside of the surface current (in the case of r > R) is filled with a substance having the specific conductivity σ' and the magnetic permeability μ'. We then have

$$\frac{H_0}{A_0}=\frac{1}{\bar{\alpha}\left\{\frac{\mu\,\bar{\alpha}I_1(\bar{\omega}_i)}{\bar{\omega}_i I_0(\bar{\omega}_i)}\left[\frac{\bar{\omega}'_i K_0(\bar{\omega}'_i)S_1}{\mu'\bar{\alpha}'K_1(\bar{\omega}'_i)}-S_2\right]-\left[\frac{\bar{\omega}'_i K_0(\bar{\omega}'_i)S_3}{\mu'\bar{\alpha}'K_1(\bar{\omega}'_i)}-S_4\right]\right\}}, \tag{19}$$

where

$$\begin{cases} S_1 = I_0(\bar{\alpha})K_1(\bar{\alpha}') + K_0(\bar{\alpha})I_1(\bar{\alpha}'); \\ S_2 = I_0(\bar{\alpha})K_0(\bar{\alpha}') - K_0(\bar{\alpha})I_0(\bar{\alpha}'); \\ S_3 = I_1(\bar{\alpha})K_1(\bar{\alpha}') - K_1(\bar{\alpha})I_1(\bar{\alpha}'); \\ S_4 = I_1(\bar{\alpha})K_0(\bar{\alpha}') + K_1(\bar{\alpha})I_0(\bar{\alpha}'). \end{cases} \tag{20}$$

Here we have

$$\begin{gathered} \bar{\omega}_i' = \beta' R = \sqrt{\frac{\sqrt{\bar{\omega}'^2 + \bar{\alpha}'^4} + \bar{\alpha}'^2}{2}} + i\sqrt{\frac{\sqrt{\bar{\omega}'^2 + \bar{\alpha}'^4} - \bar{\alpha}'^2}{2}}; \\ \bar{\omega}' = \sigma'\mu'\omega R^2; \quad \bar{\alpha}' = \alpha R; \\ \bar{\mu} = \frac{\mu}{\mu_0}; \quad \bar{\mu}' = \frac{\mu'}{\mu_0}. \end{gathered} \tag{21}$$

Substituting the value of H_0 from (19) in the expression for $\bar{F}$, we must select the modulus $|H_0|$. In practice, we are primarily interested in two cases: when the space $r > R$ is filled by a substance with $\mu' = \infty$ and $\omega' = 0$. /122 This occurs in an approximate manner when there is an iron magnetic circuit in the inductor, and when $\mu' = \mu_0$, $\sigma' = 0$ (inductor without a magnetic circuit). In these special cases, the number of variables is reduced to 3, and it is possible to express them graphically.

First case. $\mu' = \infty$, $\sigma' = 0$ outside of the inductor. Formula (19) changes into

$$\frac{H_0}{A_0} = \frac{1}{\bar{\alpha}\left[-\frac{\bar{\alpha}I_1(\bar{\omega}_i)}{\bar{\omega}_i I_0(\bar{\omega}_i)}S_2 + S_4\right]}. \tag{22}$$

Second case. $\mu' = \mu_0$, $\sigma' = 0$ outside of the inductor. Formula (19) changes into

$$\frac{H_0}{A_0} = \frac{1}{\bar{\alpha}\left\{\frac{\bar{\alpha}I_1(\bar{\omega}_i)}{\bar{\omega}_i I_0(\bar{\omega}_i)}\left[\frac{K_0(\bar{\alpha}')S_1}{K_1(\bar{\alpha}')} - S_2\right] - \left[\frac{K_0(\bar{\alpha}')S_3}{K_1(\bar{\alpha}')} - S_4\right]\right\}}. \tag{23}$$

Introducing the condition $\bar{R} = R/\rho$, which is more convenient in practice, instead of $\bar{\alpha}'$ (then $\bar{\alpha}' = \bar{\alpha}\bar{R}$), (22) and (23) may be graphically illustrated as is shown in Figure 5, a and b. Due to the fact that these functions depend slightly on $\bar{\omega}$, it was possible to plot all of the curves on two graphs. We should note that the values of H_0/A_0 in the case of $\bar{\omega} = 1$ and $\bar{\omega} = 0$ differ by less than 0.1%. The study (Ref. 12) compares formula (16) with the results obtained experimentally. Measurements were performed with cylinders having a different length. It was found that the force acting upon a unit length is greater for a short cylinder than it is for a long cylinder, in the limiting case. Extrapolating the results obtained to an infinitely long cylinder, there

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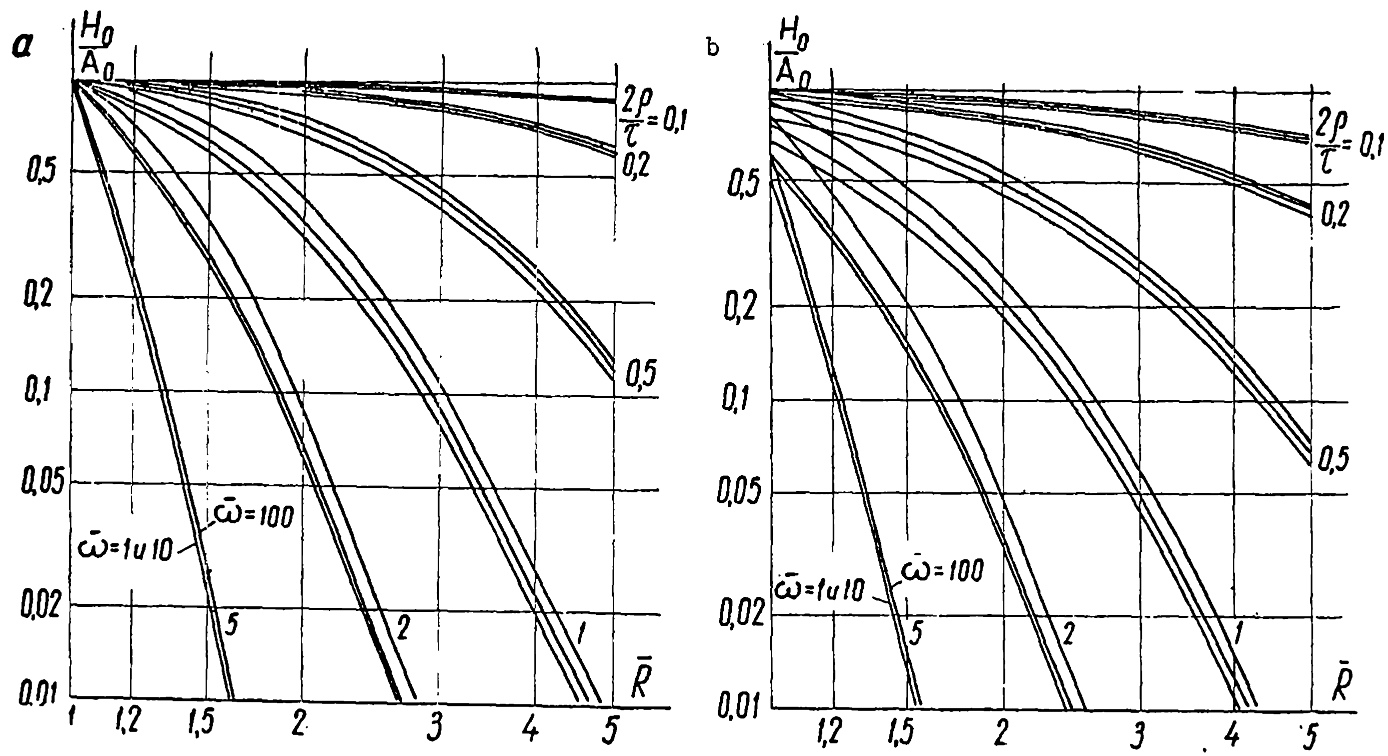

Figure 5

Dependence of H_0/A_0 on $\bar{R} = R/\rho$ for Different Values of $\bar{\omega}$ and $2\rho/\tau$: a - If $\mu' = \infty$, $\sigma' = 0$ Outside of the Inductor; b - If $\mu' = \mu_0$, $\sigma' = 0$ Outside of the Inductor. In those Sets of Curves Where the Value of $\bar{\omega}$ Is Not Shown, the Lower Curve Corresponds to the Value $\bar{\omega} = 1$; The Middle Curve Corresponds to 10, and the Upper Curve Corresponds to 100.

was satisfactory agreement with the theoretical value of (16).

6. Hollow Conductive Cylinder in the Field of an Ideal Inductor

A problem of this type was first studied in the article by I. A. Tyutin (Ref. 18), which was already mentioned. A fairly complex problem was solved: it was assumed that the cylinder is located in the field of a composite inductor, consisting of two coaxial surfaces. Currents flow along the surfaces, and one of them is located outside of the conductive cylinder, while the other is located within the conductive cylinder. However, this solution was limited to only describing general formulas.

The study (Ref. 12) derived a formula for computing the force acting upon a hollow cylinder with a wall thickness d (outer radius r_2, inner radius r_1). The inner space of the cylinder is filled with a non-conductive, and non-magnetic medium. Taking into account the given z-component of the magnetic field strength on the outer cylinder surface, we obtain the expression for the force: /123

$$\overline{F}=\frac{F\,\mathrm{cr}}{\mu H_0^2}=$$

$$=\frac{2\overline{\alpha}\,\mathrm{Im}\,\{\overline{\omega}_i\,[\overline{\alpha}\,\overline{\mu}I_0(x)T_1-\overline{\omega}_iI_1(x)T_2]\,[\overline{\alpha}\,\overline{\mu}I_0(x)T_3-\overline{\omega}_iI_1(x)T_4]^*\}}{(2-\overline{d})\,|\overline{\omega}_i\,[\overline{\alpha}\,\mu I_0(x)T_1-\overline{\omega}_iI_1(x)T_2]\,|^2}, \tag{24}$$

where $x = \alpha(1 - d)$;

$$\begin{cases} T_1=I_0(\overline{\omega}_i)K_1[\overline{\omega}_i(1-\overline{d})]+K_0(\overline{\omega}_i)I_1[\overline{\omega}_i(1-\overline{d})];\\ T_2=I_0[\overline{\omega}_i(1-\overline{d})]K_0(\overline{\omega}_i)-K_0[\overline{\omega}_i(1-\overline{d})]I_0(\overline{\omega}_i);\\ T_3=I_1(\overline{\omega}_i)K_1[\overline{\omega}_i(1-\overline{d})]-K_1(\overline{\omega}_i)I_1[\overline{\omega}_i(1-\overline{d})];\\ T_4=I_0[\overline{\omega}_i(1-\overline{d})]K_1(\overline{\omega}_i)+K_0[\overline{\omega}_i(1-\overline{d})]I_1(\overline{\omega}_i); \end{cases} \tag{25}$$

$\overline{\omega}_i$ is expressed by means of $\overline{\omega}$ and $\overline{\alpha}$ with the same relationship (11), if the outer cylinder radius is substituted in $\overline{\omega}$ and $\overline{\alpha}$:

$$\overline{\omega}=\sigma\mu\omega r_2^2;\quad \overline{\alpha}=\alpha r_2;\quad \overline{d}=\frac{d}{r_2}. \tag{26}$$

In this case, F_{cr} designates the force acting upon a volume of substance having 2d units of volume. If we set $r_1 = 0$, $r_2 = \rho$, it is natural that these formulas change into a formula for a solid cylinder.

The study (Ref. 12) also presented formulas for computing the field strengths, if it is assumed that the tangential component of the magnetic field strength on the inductor surface is given. They are quite complex, and we shall not present them here.

The works (Ref. 5, 15) presented the results derived from a small number

of experiments, in which they measured the force acting upon a hollow conductive cylinder. They did not compare the results with theory.

The results derived from a small series of measurements with respect to the force acting upon hollow cylinders were presented in (Ref. 4), but they were compared with a theoretical formula for a plane inductor.

Finally, if the thickness of the cylinder wall is small as compared with its radius, it is possible to employ the formulas for a plane layer (Ref. 21), instead of the above formulas, without entailing any large amount of error (see the article by A. Veze and L. Ulmanis in the present collection).

7. Solid Cylinder Having Finite Length in the Field of an Ideal Inductor

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The theory presented above pertained to cases when it was assumed that the conductive cylinder had an infinite length -- i.e., the influence of the finite length of the cylinder was disregarded.

G. Kh. Kirshteyn (Ref. 9) discovered a method for solving a similar problem for a solid cylinder having finite length, on the assumption that there is no air gap between the cylinder and the inductor and that the space behind the inductor has infinite magnetic permeability.

This method is as follows.

Let the cylinder have the length 2ℓ (Figure 6). Let us divide all of the space within the inductor into three regions: $z \leqslant -\ell$ (region I); $-\ell \leqslant z \leqslant \ell$ (region II); $z \geqslant \ell$ (region III).

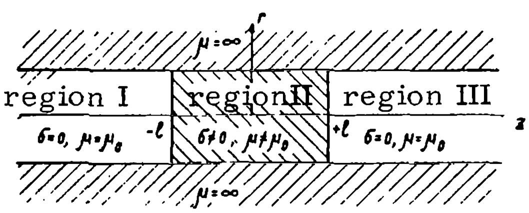

Figure 6

Arrangement of Conductive Cylinder Having Finite Length in Inductor

In each of these regions separately, we shall try to find the magnetic field in the form of the sum of two fields H_1 and H_2, where H_1 is the field of a hollow inductor (1b) in the regions I and III, while in region II it is the field of the inductor filled with a uniform, conductive substance having the characteristics of the finite cylinder under consideration, which is described by expressions (1a).

H_2 is the perturbation of the field H_1 close to the ends of the cylinder. It thus follows that in the case of $z \to \pm\infty$ the field H_2 must strive to zero. It follows from the condition that $\mu = \infty$ outside of the inductor that the z-component of the magnetic field is the same over the entire surface of the inductor, no matter whether there is a cylinder there or not. As is customary, it is assumed that this component equals $H_0 e^{i(\omega t-\alpha z)} = A_0 e^{i(\omega t-\alpha z)}$. However, since this condition is fulfilled

by the field $\mathbf{H}_1$, it follows that the tangential component of $\mathbf{H}_2$ on the inductor surface equals zero. This is the boundary condition for computing $\mathbf{H}_2$.

The following solution satisfies these requirements:

In region I ($z \leqslant -\ell$) /125

$$\begin{cases} H_{r2}{}^{\mathrm{I}} = -\dfrac{i}{\omega\mu_0}\displaystyle\sum_{n=1}^{\infty} C_{1n} b_{an} J_1\left(\frac{x_{0n} r}{R}\right) e^{b_{an} z} e^{i\omega t}; \\ H_{z2}{}^{\mathrm{I}} = \dfrac{i}{\omega\mu_0 R}\displaystyle\sum_{n=1}^{\infty} C_{1n} x_{0n} J_0\left(\frac{x_{0n} r}{R}\right) e^{b_{an} z} e^{i\omega t}; \\ E_{\varphi 2}{}^{\mathrm{I}} = \displaystyle\sum_{n=1}^{\infty} C_{1n} J_1\left(\frac{x_{0n} r}{R}\right) e^{b_{an} z} e^{i\omega t}; \end{cases} \tag{27a}$$

in region II ($-\ell \leqslant z \leqslant \ell$)

$$\begin{cases} H_{r2}{}^{\mathrm{II}} = -\dfrac{i}{\omega\mu}\displaystyle\sum_{n=1}^{\infty} b_{in} J_1\left(\frac{x_{0n} r}{R}\right)\left(C_{2n} e^{b_{in} z} - C_{3n} e^{-b_{in} z}\right) e^{i\omega t}; \\ H_{z2}{}^{\mathrm{II}} = \dfrac{i}{\omega\mu R}\displaystyle\sum_{n=1}^{\infty} x_{0n} J_0\left(\frac{x_{0n} r}{R}\right)\left(C_{2n} e^{b_{in} z} + C_{3n} e^{-b_{in} z}\right) e^{i\omega t}; \\ E_{\varphi 2}{}^{\mathrm{II}} = \displaystyle\sum_{n=1}^{\infty} J_1\left(\frac{x_{0n} r}{R}\right)\left(C_{2n} e^{b_{in} z} + C_{3n} e^{-b_{in} z}\right) e^{i\omega t}; \end{cases} \tag{27b}$$

in region III ($z \geqslant \ell$)

$$\begin{cases} H_{r2}{}^{\mathrm{III}} = \dfrac{i}{\omega\mu_0}\displaystyle\sum_{n=1}^{\infty} C_{4n} b_{an} J_1\left(\frac{x_{0n} r}{R}\right) e^{-b_{an} z} e^{i\omega t}; \\ H_{z2}{}^{\mathrm{III}} = \dfrac{i}{\omega\mu_0 R}\displaystyle\sum_{n=1}^{\infty} C_{4n} x_{0n} J_0\left(\frac{x_{0n} r}{R}\right) e^{-b_{an} z} e^{i\omega t}; \\ E_{\varphi 2}{}^{\mathrm{III}} = \displaystyle\sum_{n=1}^{\infty} C_{4n} J_1\left(\frac{x_{0n} r}{R}\right) e^{-b_{an} z} e^{i\omega t}. \end{cases} \tag{27c}$$

Here

$$b_{in} = \sqrt{\frac{x_{0n}^2}{R^2} - i\sigma\mu\omega}; \quad b_{an} = \frac{x_{0n}}{R}, \tag{28}$$

where x_{on} are the roots of the Bessel function of the first kind of zero order ($J_0(x_{on}) = 0$); R is the radius of the inductor and the cylinder; $C_{1n}, \ldots, C_{4n}$ /126

are the integration constants, which may be determined from the boundary conditions (for the total field)

$$\left.\begin{aligned}\mu_0 H_z^{\mathrm{I}} &= \mu H_z^{\mathrm{II}}\\ H_r^{\mathrm{I}} &= H_r^{\mathrm{II}}\\ E_\varphi^{\mathrm{I}} &= E_\varphi^{\mathrm{II}}\end{aligned}\right\}\ \text{for}\ z=-l;\qquad \left.\begin{aligned}\mu H_z^{\mathrm{II}} &= \mu_0 H_z^{\mathrm{III}}\\ H_r^{\mathrm{II}} &= H_r^{\mathrm{III}}\\ E_\varphi^{\mathrm{II}} &= E_\varphi^{\mathrm{III}}\end{aligned}\right\}\ \text{for}\ z=l. \tag{29}$$

The first and third conditions of (29) yield identical equations. Thus, four equations are obtained with four unknowns for each n, from which the constants may be determined.

The study (Ref. 9) found the expression for the perturbation of the force caused by the presence of the cylinder ends:

$$\begin{aligned}\Delta\bar{F} = -\operatorname{Im}\Bigg\{\frac{2\bar{\alpha}^2\bar{\omega}^2}{k}\sum_{n=1}^{\infty}\frac{1}{(x_{0n}^2+\bar{\alpha}^2)(p_n^2+\bar{\alpha}^2)}\times\\ \times\Bigg[\frac{x_{0n}\sin\bar{\alpha}k+\bar{\alpha}\cos\bar{\alpha}k}{p_n \operatorname{ch} p_n k + x_{0n}\operatorname{sh} p_n k}\left(p_n \operatorname{ch} p_n k \sin\bar{\alpha}k - \bar{\alpha}\operatorname{sh} p_n k\cos\bar{\alpha}k\right)+\\ +\frac{x_{0n}\cos\bar{\alpha}k-\bar{\alpha}\sin\bar{\alpha}k}{p_n \operatorname{sh} p_n k + x_{0n}\operatorname{ch} p_n k}\left(p_n \operatorname{sh} p_n k\cos\bar{\alpha}k+\bar{\alpha}\operatorname{ch} p_n k\sin\bar{\alpha}k\right)\Bigg]\Bigg\}.\end{aligned} \tag{30}$$

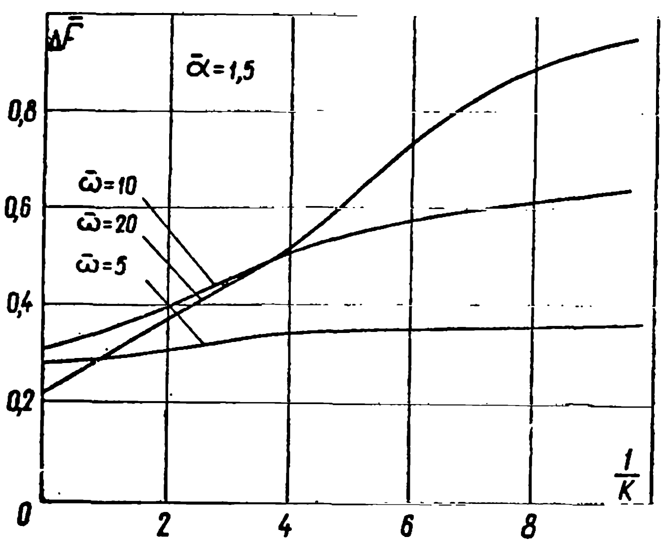

Figure 7

Dependence of $\Delta\bar{F}$ on $\bar{k}$ in the Case of $\bar{\alpha}$ = 1.5 for Certain Values of $\bar{\omega}$

Here /128

$$p_n=\sqrt{x_{0n}^2+i\bar{\omega}};\quad k=\frac{l}{R};\quad \Delta\bar{F}=\frac{\bar{\alpha}\Delta F}{2\pi\mu_0 A_0^2 l}. \tag{31}$$

ΔF designates the difference between the total force acting upon a finite cylinder, and the force acting upon a segment having the same length of an infinitely long cylinder.

In the case of k>>0.4, this formula may be simplified as follows:

$$\Delta\bar{F} \approx \frac{2\bar{\alpha}^2\bar{\omega}}{k} \sum_{n=1}^{\infty} \frac{\left(x_{0n} - \frac{u_n}{2}\right)[(x^2_{0n}+\bar{\alpha}^2)^2 - \bar{\omega}^2] + 2\bar{\omega}^2\left[x_{0n} - \frac{x_{0n}^2+\bar{\alpha}^2}{\sqrt{2}\,u_n}\right]}{(x^2_{0n}+\bar{\alpha}^2)[(x^2_{0n}+\bar{\alpha}^2)^2+\bar{\omega}^2]^2}; \tag{32a}$$

$$u_n = \sqrt{\sqrt{\bar{\omega}^2 + x^4_{0n}} + x^2_{0n}}.$$

Figures 7 and 8, a and b, present the graphic dependence of $\Delta\bar{F}$ for certain values of $\bar{\omega}$, $\bar{\alpha}$ and k computed according to formula (30). It should also be noted that, for a sufficiently large value of $\bar{\alpha}$, $\Delta\bar{F}$ may be negative for certain values of k.

A comparison of the results obtained with the experimental data existing previously (Ref. 12) shows satisfactory agreement.

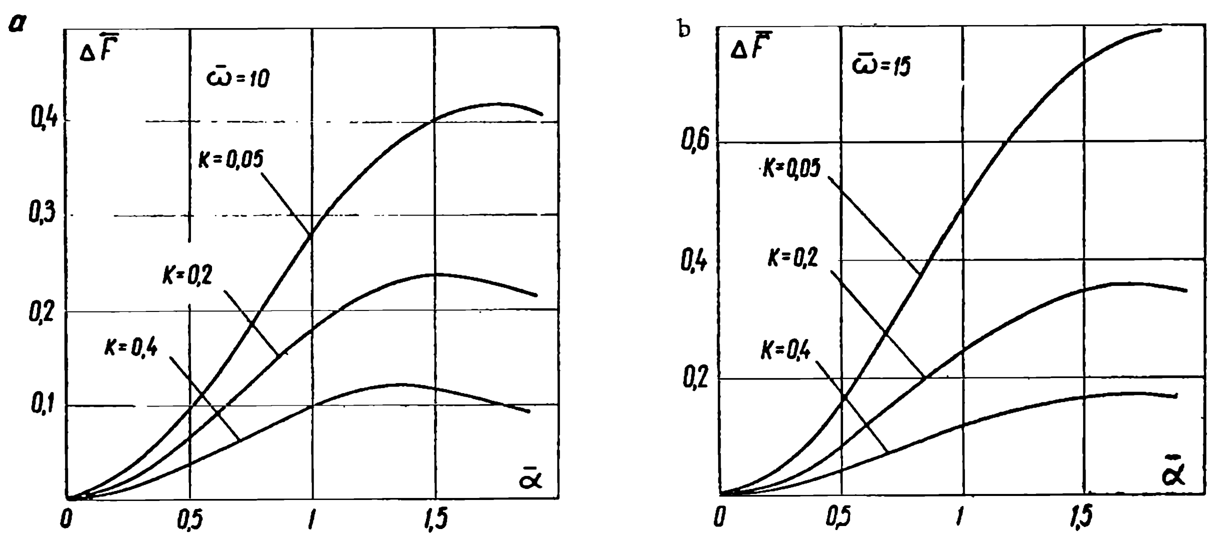

Figure 8

Dependence of $\Delta\bar{F}$ on $\bar{\alpha}$ for Certain Values of $\bar{k}$: a - In the case of $\bar{\omega}$ = 10; b - In the case of $\bar{\omega}$ = 15 /127

However, it must be stated that this problem must still be investigated in greater detail.

8. Conductive Cylinder in the Field of a Real Inductor

U. A. Saulite, A. E. Mikel'son et. al. have studied the problem of the behavior of an infinitely long cylinder in the field of a real inductor, when

this inductor approximated the model which was discussed in Chapter 2 (Ref. 16, 17). In this formulation, the problem quite accurately reflects the case in which the cylinder ends project sufficiently far from the inductor ends, where the field of the inductor has decreased almost to zero.

It was shown in (Ref. 15) that, if the cylinder is length is small as compared with the inductor length, the force does not remain constant when the cylinder is displaced along the axis of the inductor. The force can even change sign at the end of it (Figure 9). The experiments were carried out with aluminum cylinders.

A calculation was made in (Ref. 17) of the radial component of the magnetic field strength in a system consisting of a real inductor -- whose model is described in Chapter 2 -- and an infinitely long cylinder placed in it (Figure 10). It was found that H_r is expressed in dimensionless form as follows, with- /129
in the conductive cylinder

$$\bar{H}_{r1}=\frac{2\pi H_{r1}}{\mu_0 A_0}=2\int_0^\infty \frac{K_1(\bar{\lambda})}{J_1(\bar{k}\bar{r}_1)}\Bigg[I_1(\bar{\lambda}\bar{r}_1)+$$

$$+\frac{J_1(\bar{k}\bar{r}_1)\bar{\lambda}I_0(\bar{\lambda}\bar{r}_1)\bar{\mu}-\bar{k}J_0(\bar{k}\bar{r}_1)I_1(\bar{\lambda}\bar{r}_1)}{J_1(\bar{k}\bar{r}_1)\bar{\lambda}K_0(\bar{\lambda}\bar{r}_1)\bar{\mu}+\bar{k}J_0(\bar{k}\bar{r}_1)K_1(\bar{\lambda}\bar{r}_1)}K_1(\bar{\lambda}\bar{r}_1)\Bigg]\times \tag{33a}$$

$$\times\sum_{n=1}^{S}\left\{\cos\bar{\lambda}\left(\frac{n\bar{\tau}}{3}-\bar{z}\right)-\cos\bar{\lambda}\left[\frac{(n-1)\bar{\tau}}{3}-\bar{z}\right]\right\}J_1(\bar{k}\bar{r})e^{-i\left[\omega t+\frac{(n-1)\pi}{3}\right]}d\bar{\lambda},$$

outside of the conductive cylinder in the case of $\rho \leqslant r \leqslant R$ /130

$$\bar{H}_{r2}=\frac{2\pi H_{r2}}{\mu_0 A_0}=2\int_0^\infty K_1(\bar{\lambda})\frac{J_1(\bar{k}\bar{r}_1)\bar{\lambda}I_0(\bar{\lambda}\bar{r}_1)\bar{\mu}-\bar{k}J_0(\bar{k}\bar{r}_1)I_1(\bar{\lambda}\bar{r}_1)}{J_1(\bar{k}\bar{r}_1)\bar{\lambda}K_0(\bar{\lambda}\bar{r}_1)\bar{\mu}+\bar{k}J_0(\bar{k}\bar{r}_1)K_1(\bar{\lambda}\bar{r}_1)}\times$$

$$\times\sum_{n=1}^{S}\left\{\cos\bar{\lambda}\left(\frac{n\bar{\tau}}{3}-\bar{z}\right)-\cos\bar{\lambda}\left[\frac{(n-1)\bar{\tau}}{3}-\bar{z}\right]\right\}K_1(\bar{\lambda}\bar{r})e^{-i\left[\omega t+\frac{(n-1)\pi}{3}\right]}d\bar{\lambda}+ \tag{33b}$$

$$+\sum_{n=1}^{S}\sqrt{\frac{1}{\bar{r}}}\left\{\left[\left(\frac{2}{k_1}-k_1\right)K(k_1)-\frac{2}{k_1}E(k_1)\right]-\right.$$

$$\left.-\left[\left(\frac{2}{k_2}-k_2\right)K(k_2)-\frac{2}{k_2}E(k_2)\right]\right\}e^{-i\left[\omega t+\frac{(n-1)\pi}{3}\right]}.$$

The notation employed here coincides with the notation in formula (3). In addition,

$$\bar{\lambda}=\lambda R;\quad \bar{k}=\sqrt{\bar{\lambda}^2-i\omega\bar{r}^2};\quad \bar{r}_1=\frac{\rho}{R};\quad \bar{\mu}=\frac{\mu}{\mu_0}; \tag{34}$$

where ρ is the radius of the conductive cylinder.

The report in question does not present the expressions for the tangential

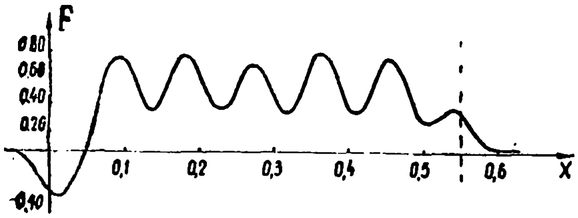

Figure 9

Change in the Force Acting Upon a Cylinder Along the Inductor Axis; (F - in newtons; x - in meters, τ = 0.273 m)

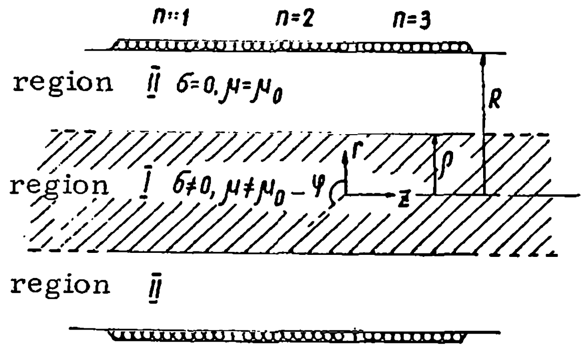

Figure 10

Mutual Arrangement of an Inductor Having Finite Length and a Conductive Cylinder

component of the magnetic field, nor the expression for the electric field strength. The forces acting upon the cylinder have still not been computed.

9. Conductive Sphere in the Field of a Cylindrical Inductor

A sphere, as a body with the greatest symmetry, occupies a special position among all bodies having finite dimensions. Therefore, it is definite interest to determine the ponderomotive forces on both a solid and hollow sphere. The author of this article attempted to calculate the forces acting upon a solid sphere located in the field of an ideal, cylindrical inductor, under the assumption that $\mu = \infty$ outside of the inductor (Ref. 10, 11). However, in formal terms the solution obtained is not suitable for the computations.

Experiments measuring the forces acting upon spheres have shown that the dependence of the force change upon $\bar{\omega}$ [$\bar{\omega} = \sigma\mu\omega\rho^2$, ρ is the radius of the sphere if the sphere is solid, and $\bar{\omega} = \sigma\mu\omega b^2$ (b is the thickness of the sphere wall) if the sphere is hollow] is exactly the same as in all other cases. For small $\bar{\omega}$ the force is directly proportional to $\bar{\omega}$. A maximum is then reached, and when $\bar{\omega}$ increases, it begins to decrease.

Figure 11, a, b and c, presents the results derived from measurements /132 performed with spheres having a different radius and made of a different material -- copper, aluminum, tin, and lead. These spheres were placed in an inductor without a magnetic circuit (the quantity $F/A_0^2\rho^2$, which is a dimensional quantity, is plotted along the ordinate axis. However, in this case this is of no value). In the case when α = 0.31, it was found that the ordinates of all four curves were proportional to the radius of the sphere. Therefore, if we introduce the condition $F/A_0^2\rho^3$, they may all be combined on one curve [see /133

/131

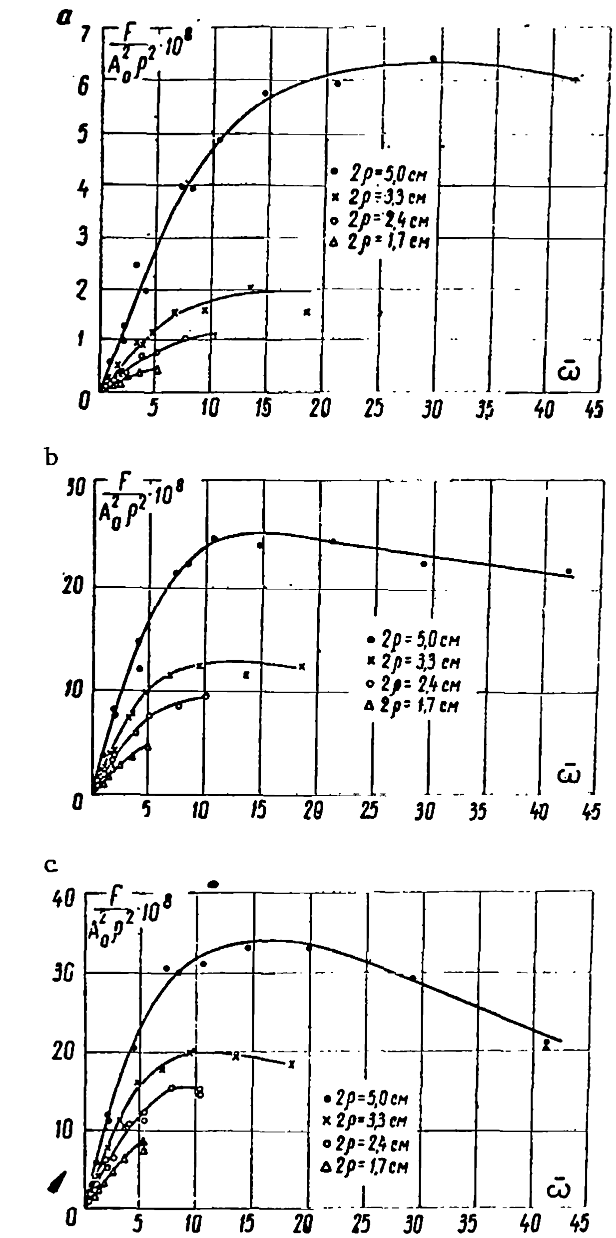

Figure 11

Dependence of $F/A_0{}^2\rho^2$ on $\bar{\omega}$ for Solid Spheres Having Certain Radii: a - in the case of $\alpha = 0.93$; b - in the case of $\alpha = 0.62$; c - in the case of $\alpha = 0.31$. The Inductor Radius is R = 4.0 cm; F is Given in Newtons; A_0 is Given in a/m; ρ is Given in Meters.

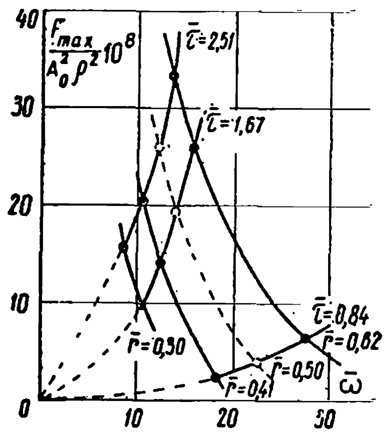

Figure 12

Dependence of the Maximum Force on $\bar{r}$ and $\bar{\tau}$ for Solid Spheres.

F_{max} is Given in Newtons; A_0 - in a/m; ρ - in Meters; $\bar{\tau} = \tau/R$; $r = \rho/R$.

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also (Ref. 11)]. However, for other values of α, this did not occur. This indicates that the above dependence is not universally valid.

The maximum force is of great importance in designing different electromagnetic carriers. The study (Ref. 5) presents a graph which makes it possible to determine the value of $F_{max}/A_0{}^2\rho^2$ from $\bar{\tau}$, $\bar{r}$ and $\bar{\omega}$, where $\bar{\tau} = \tau/R$, and $\bar{r} = \rho/R$ (R is the inductor radius) (Figure 12). This graph was compiled on the basis of experimental results, and is incomplete in the sense that it does not encompass a large enough range of possible parameter changes.

The results derived from measuring the ponderomotive forces acting upon a hollow sphere (Figure 13) were also presented in (Ref. 5). $\bar{F}' = F/A_0{}^2b^2$, $\bar{\omega}' = \sigma\mu\omega b^2$; b is the thickness of the sphere wall.

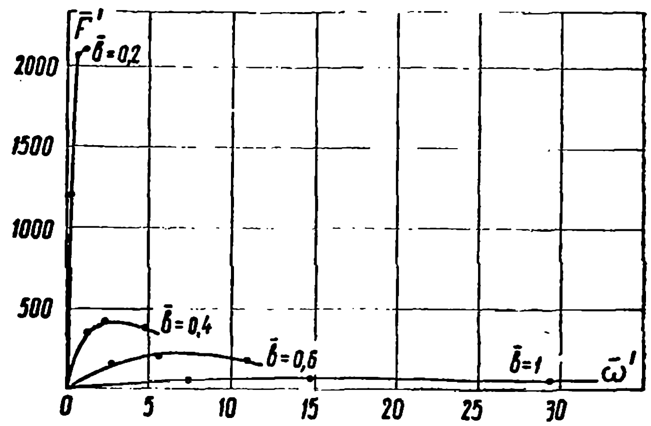

Figure 13

Dependence of $\bar{F}'$ on $\bar{\omega}'$ for Certain Hollow Spheres

We shall refrain from discussing the results in greater detail, since it is expected that a suitable method for computing the forces acting upon a sphere will be found in the very near future.

REFERENCES

1. Blake, R. Improvements Relating to Electromagnetic Pumps. Patent No. 8323/56. Index at acceptance: Clfcc 35, A. IX International classification HO2k.
2. Veze, A.K., Mikel'son, A.E. Issledovaniye vozmozhnosti perekachivaniya zhidkikh metallov s pomoshch'yu tsilindricheskikh nasosov bez ferromagnitnogo serdechnika (Study of Possible Pumping of Molten Metals by Means of Cylindrical Pumps without a Ferromagnetic core). V Kn: Voprosy magnitnoy gidrodinamiki (In the Book: Problems of Magnetic Hydrodynamics), 3. Izdatel'stvo AN Latv. SSR, 1963.
3. Grigor'yev, M.N. Eksperimental'noye issledovaniye magnitnykh poley tsilindricheskikh induktorov (Experimental Study of Magnetic Fields of Cylindrical Inductors). Ibid, 1953.
4. Grigor'yev, M.N. Eksperimental'noye issledovaniye ponderomotornykh sil v lineynykh tsilindricheskikh induktorakh (Experimental Study of Ponderomotive Forces in Linear Cylindrical Inductors). Ibid, 1963.
5. Dobrayakov, D., Krumin', Yu., Klyavin', Ya., Nikolayev, V. Issledovaniye vozmozhnostey transportirovki sfericheskikh provodyashchikh tel begushchim magnitnym polem (Study of the Possible Transporting of Spherical Conducting Media with a Traveling Magnetic Field). Izvestiya AN Latv. SSR, 12, 1961.
6. Dobryakov, D.D., Kirko, I.M. Elektromagnitnaya transportirovka konteynerov (Electromagnetic Transporting of Containers). Atomnaya energiya, 1, 1962.
7. Dobryakov, D., Nikolayev, V., Saulite, U. Ustroystvo elektromagnitnoy

pochty na atomnom reaktore (Arrangement of an Electromagnetic Rabbit in an Atomic Reactor). Izvestiya AN Latv. SSR, 2, 1963.

8. Kirko, I.M. Kriterii podobiya elektrodinamicheskikh yavleniy pri otnositel'nom dvizhenii magnitnogo polya i provodyashchey sredy (Dimensionless Numbers of Electrodynamic Phenomena in the Case of Relative Motion of a Magnetic Field and a Conductive Medium). V Kn: Voprosy energetiki (In the Book: Problems of Energetics), 3. Izdatel'stvo AN Latv. SSR, 1955.
9. Kirshteyn, G.Kh. Provodyashchiy tsilindr konechnoy dliny, pomeshchennyy v begushcheye magnitnoye pole tsilindricheskogo ferromagnitnong induktora (Conductive Cylinder of Finite Length Placed in a Traveling Magnetic Field of a Cylindrical, Ferromagnetic Inductor). Izvestiya AN Latv. SSR, Seriya Fizicheskikh i Tekhnicheskikh Nauk, 3, 1965.
10. Krumin', Yu.K. Zadacha o provodyashchem share, nakhodyashchemsya v begushchem magnitnom pole (Problem of a Conductive Sphere Placed in a Traveling Magnetic Field). V Kn: Prikladnaya magnitogidrodinamika (In the Book: Applied Magnetohydrodynamics). Izdatel'stvo AN Latv. SSR, 1956.
11. Krumin', Yu.K. Zadacha o provodyashchem share, nakhodyashchemsya v begushchem magnitnom pole (Problem of a Conductive Sphere Located in a Traveling Magnetic Field). Izvestiya AN Latv. SSR, 5, 1957. /134
12. Krumin', Yu.K. Zadacha o provodyashchem tsilindre, nakhodyashchemsya v begushchem magnitnom pole tsilindricheskogo induktora (Problem of a Conductive Cylinder Located in a Traveling Magnetic Field of a Cylindrical Inductor). V Kn: Elektromagnitnyye protsessy v metallakh (In the Book: Electromagnetic Processes in Metals). Izdatel'stvo AN Latv. SSR, 1959.
13. Drumin', Yu.K. Vychisleniye ponderomotornykh sil, deystvuyushchikh na provodyashchiy tsilindr v begushchem magnitnom pole tsilindricheskogo induktora (Computation of Ponderomotive Forces Acting on a Conductive Cylinder in a Traveling Magnetic Field of a Cylindrical Inductor). Izvestiya AN Latv. SSR, Seriya Fizicheskikh i Tekhnicheskikh Nauk, 3, 1966.
14. Kunin, P.Ye., Taksar, I.M. Magnitnoye pole konechnogo solenoida (Magnetic Field of a Finite Solenoid). Izvestiya AN Latv. SSR, 9, 1951.
15. Mikel'son, A.E., Saulite, U.A., Nikolayev, V.N. Modelirovaniye sil, deystvuyushchikh na provodyaschiye tela v begushchem magnitnom pole tsilindricheskogo induktora (Simulation of Forces Acting Upon Conductive Bodies in a Traveling Magnetic Field of a Cylindrical Inductor). Izvestiya AN Latv. SSR, Seriya Fizicheskikh i Tekhnicheskikh Nauk 3, 1964.
16. Mikel'son, A.E., Nikolayev, V.N., Saulite, U.A. Opredeleniye radial'noy sostavlyayushchey begushchego magnitnogo polya v induktore tsilindricheskogo besserdechnikovogo nasosa (Determination of the Radial Component of a Traveling Magnetic Field in an Inductor of a Cylindrical Pump without a Core). Izvestiya AN Latv. SSR, Seriya Fizicheskikh i Tekhnicheskikh Nauk 5, 1964.
17. Mikel'son, A.E., Saulite, U.A., Shkerstena, A.Ya. Issledovaniye tsilindricheskikh besserdechnikovykh nasosov (Study of Cylindrical Pumps without Cores). Magnitnaya Gidrodinamika 2, 1965.
18. Tyutin, I.A. Mekhanicheskiye sily v begushchem elektromagnitnom pole (Mechanical Forces in a Traveling Electromagnetic Field). V Kn: Voprosy energetiki (In the Book: Problems of Energetics), 3. Izdatel'stvo AN Latv. SSR, 1955.

19. Tyutin, I.A. Elektromagnitnyye nasosy dlya zhidkikh metallov (Electromagnetic Pumps for Molten Metals). Riga, 1959.
20. Egle, I.Yu., Yankop, E.K. Analiticheskiy raschet elektromagnitnykh poley v induktsionnykh nasosakh tsilindricheskogo tipa (Analytical Design of Electromagnetic Fields in Induction Pumps of the Cylindrical Type). Uchenyye Zapiski Russkogo Politekhnicheskogo Instituta, 3, 7, 1962.
21. Egle, I.Yu., Yankop, E.K. Pogreshnost' rascheta davleniya v induktsionnykh tsilindricheskikh nasosakh pri ispol'zovanii formul ploskikh nasosov (Error in Computing Pressure in Induction Cylindrical Pumps when Employing Formulas for Plane Pumps). Uchenyye Zapiski Russkogo Politekhnicheskogo Instituta, 4, 9, 1963.
22. Yankop, E.K. Raspredeleniye skorosti potoka i magnitogidrodinamicheskiye poteri davleniya v gorlovine koaksial'nogo induktsionnogo nasosa (Velocity Distribution of Current and Magnetohydrodynamic Losses of Pressure in the Entrance of a Coaxial Induction Pump). Uchenyye Zapiski Leningradskogo Gosudarstvennogo Universiteta, 10, 1957.

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THEORY FOR THE PROPAGATION OF PULSED ELECTROMAGNETIC FIELDS IN MOVING CONDUCTIVE MEDIA

G. Ya. Sermons

1. Introduction

The necessity of developing a theory for the propagation of electromagnetic pulsed fields in conductive media has arisen in the last twenty years due to the utilization of relaxation processes in ferromagnets (Ref. 1). This also pertains to the necessity of studying the influence of transitional processes which occur in electrotransmission lines when the current is switched on or off, as well as in telephone and telegraph lines (Ref. 2-3). A theory for the propagation of electromagnetic pulsed fields has been developed intensely during the last 50-60 years due to the use of pulsed fields for geophysical research (Ref. 4-22). The essence of the so-called method for establishing a current (Ref. 23), which is used for geological exploration, consists of establishing an electric or magnetic field on the surface of the earth when a constant current is switched to a transmitting antenna. A horizontal, grounded conductor or loop is used as the transmitting antenna. In many cases, the distance from the receiving antenna to the transmitting antenna is large, as compared with the dimensions of the transmitting antenna. The transmitting antenna may be regarded as an electric (horizontal conductor) or magnetic (loop) dipole.

Studies on the propagation of an electromagnetic field impulse in conductive media may be reduced to investigating the formation of the field of an electric or magnetic dipole which is located in a conductive medium (Ref. 2, 6), in a uniform conductive half-space (Ref. 4, 13, 20, 21), or in a non-uniform, stratified medium (Ref. 7-17).

It was found long ago that electromagnetic pulsed fields may be used to develop new methods of measuring the flow rates of an electroconductive liquid

(Ref. 24-26). In these methods, pulsed currents are induced in a moving, conductive liquid by switching on the current in the primary coil. The flow rate of this liquid may be determined by changes in the time intervals of the maximum shift (Ref. 24) or the electromotive force passing through zero (Ref. 25) in the receiving coil. This is caused by the fact that the conductive liquid removes the pulsed currents. Research on the propagation of electromagnetic pulsed fields in moving, conductive media was carried out due to the necessity of making an exact determination of the dependence between the measured intervals and the velocity of the liquid (Ref. 27-29). /136

The review by V. V. Novikov (Ref. 22) presents an extensive bibliography and a survey of research on the propagation of pulsed signals in conductive media and above the earth's surface when there is no motion. In this article, we shall confine ourselves to examining the methods and results given in these articles, which may be directly applied for studying the propagation of pulsed fields in moving, electroconductive media. We shall not deal with the problem of considering the displacement currents, since they do not play a significant role in this formulation. Thus, the phenomena which we shall investigate are limited to problems which include different solutions of the Maxwell equations for moving media (Ref. 27)

$$\begin{gathered}\operatorname{rot} \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}; \\ \operatorname{rot} \mathbf{H} = \sigma \mathbf{E} + \sigma\mu_0[\mathbf{v}\mathbf{H}], \quad \operatorname{div} \mathbf{H} = 0.\end{gathered} \tag{1}$$

We previously disregarded the magnetic properties of the medium ($\mu_0 = 4\pi\ 10^{-7}$ h/m). Introducing the electromagnetic potentials

$$\mathbf{H} = \frac{1}{\mu_0} \operatorname{rot} \mathbf{A} \text{ and } \mathbf{E} = \operatorname{grad} \varphi - \frac{\partial \mathbf{A}}{\partial t},$$

we obtain

$$\nabla^2 \mathbf{A} - \sigma\mu_0 (\mathbf{v} \nabla \mathbf{A}) = \sigma\mu_0 \frac{\partial \mathbf{A}}{\partial t} \tag{2}$$

and

$$\varphi = \frac{1}{\sigma\mu_0} \operatorname{div} \mathbf{A} - \mathbf{v}\mathbf{A}.$$

2. Propagation of Pulsed Electromagnetic Fields in Conductive Media

The investigation of the propagation of an electromagnetic field impulse in conductive media is considerably simplified when the medium is not in motion. In a few of the simplest cases (the medium moves like one entire body, there is only one component of the vector potential) there is no necessity of resorting to the solution of equations for moving media (2). We may thus confine ourselves to employing the Galilean transformation (Ref. 28, 29). In this connection, it is pertinent to investigate the most characteristic methods and results of solving the problem when there is no motion -- i.e., the group of problems which include different solutions of the equations /137

$$\nabla^2 A = \sigma\mu_0 \frac{\partial A}{\partial t};$$

$$\varphi = \frac{1}{\sigma\mu_0} \operatorname{div} A \qquad (3)$$

with the corresponding boundary conditions.

Let us begin by examining the simplest case, when an electric dipole having the length dS is placed in a medium with uniform conductivity σ. The current passing through it has the form of a single inclusion function

$$\mathfrak{I}(t) = \mathfrak{I}\gamma(t),$$

where

$$\gamma(t) = \begin{cases} 0, & t<0; \\ 1, & t>0 \end{cases} \qquad (4)$$

is the unit step function of time.

The solution of this problem has been obtained by Ollendorf (Ref. 2, 3) by means of an operational method, on the basis of the well known relationship of the Hertz vector (Ref. 2) for a dipole supplied by an alternating current

$$\Pi(\omega) = -\frac{\mathfrak{I} ds}{4\pi\sigma r} e^{\sqrt{i\omega\sigma\mu_0} r}. \qquad (5)$$

It is known from the theory of the Laplace transformation (Ref. 30) that, by applying the operator L^{-1} to the constant $\mathfrak{I}$, we obtain

$$L^{-1}\mathfrak{I} = \frac{1}{2\pi i} \int_{\alpha - i\infty}^{\alpha + i\infty} \mathfrak{I} \frac{e^{pt}}{p} dp = \mathfrak{I}\gamma(t). \qquad (6)$$

Consequently, substituting $p = i\omega$ in (5) and applying the operator L to the expression (5), with allowance for the well known relationship (Ref. 30), we obtain the expression for the Hertz potential

$$\Pi(t) = \frac{\mathfrak{I} ds}{4\pi\sigma r}\left[1 - \Phi\left(\frac{1}{\sqrt{\bar{t}}}\right)\right]\gamma(t), \qquad (7)$$

where

$$\bar{t} = \frac{4t^{*}}{\sigma\mu_0 r^2}, \quad \Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du.$$

For a dipole directed along the Z axis, the relationship between the single component of the vector potential A_z and the Hertz potential has the following form /138

$$A_z = \sigma\mu_0 \Pi, \qquad (8)$$

and the electric field components may be expressed by means of the Hertz potential as

$$E_x = -\frac{\partial\varphi}{\partial x}, \quad E_y = -\frac{\partial\varphi}{\partial y}, \quad \varphi = \frac{\partial\Pi}{\partial t}, \qquad (9)$$

$$E_z=\frac{\partial^2\Pi}{\partial x^2}+\frac{\partial^2\Pi}{\partial y^2}. \quad (9)$$

Potential II satisfies the equation

$$\nabla^2\Pi=\sigma\mu_0\frac{\partial\Pi}{\partial t}. \quad (10)$$

For an infinitely long conductor directed along the Z axis (Ref. 2), $\phi = 0$ and

$$\Pi=\frac{\mathfrak{I}}{4\pi\sigma}\int_{-\infty}^{\infty}\left[1-\Phi\left(\frac{1}{\sqrt{\bar{t}}}\right)\right]\frac{d\varsigma}{r}, \quad (11)$$

where $r=\sqrt{\rho^2+\varsigma^2}$; ρ is the distance from the observation point up to the axis; ς is the distance from the observation point to the dipole. In this case, only one component of the electric field remains

$$E_z=\sigma\mu_0\frac{\partial\Pi}{\partial t}=\frac{\sigma\mu_0\mathfrak{I}}{4\pi}\cdot\int_{-\infty}^{\infty}\frac{\partial}{\partial t}\left[1-\Phi\left(\frac{1}{\sqrt{\bar{t}}}\right)\right]\frac{d\varsigma}{\sqrt{\rho^2+\varsigma^2}}=$$
$$=\frac{\mathfrak{I}}{\pi\rho^2\sigma}\cdot\frac{1}{\bar{t}}e^{-\frac{1}{\bar{t}}}. \quad (12)$$

Figure 1 presents the dependence of $\bar{E}_z=\frac{\pi\rho^2\sigma}{\mathfrak{I}}E_z$ on $\bar{t}$.

A. N. Tikhonov (Ref. 4) first obtained the most complete solution of the problem for the same dipole placed on the surface of a uniform, conductive half-space. A. N. Tikhonov and O. A. Skugarevskaya also solved the problem for the dipole placed in a non-uniform, stratified medium (Ref. 7-12, 15-17). The solution of all these problems is based on the solution of the thermal conductivity equation for the second and third boundary value problems. The initial equations with partial separation of the variables are reduced to the one-dimensional case. The use of the two methods -- the reflection and /139 the Fourier method -- yielded the dependence of the electromagnetic field over a wide range of changes in time t.

We shall try to present the main results of these studies. Let us first examine the case when an electric dipole, which is placed at the origin, is directed along the Y axis and is located at the boundary z = 0 of a uniform, conductivity half-space. Just as previously, the current changes according to the unit step function (4), which leads to zero initial conditions. This orientation of the dipole makes it possible to set $A_x = 0$, and the boundary conditions for the remaining components of the vector potential and the scalar potential may be reduced to the following equations of continuity:

$$A_{yI}|_{z=0}=A_{yII}|_{z=0},\quad A_{zI}|_{z=0}=A_{zII}|_{z=0},$$
$$\left.\frac{\partial A_{yI}}{\partial z}\right|_{z=0}=\left.\frac{\partial A_{yII}}{\partial z}\right|_{z=1} \text{ и } \varphi_I|_{z=0}=\varphi_{II}|_{z=0}, \quad (13)$$

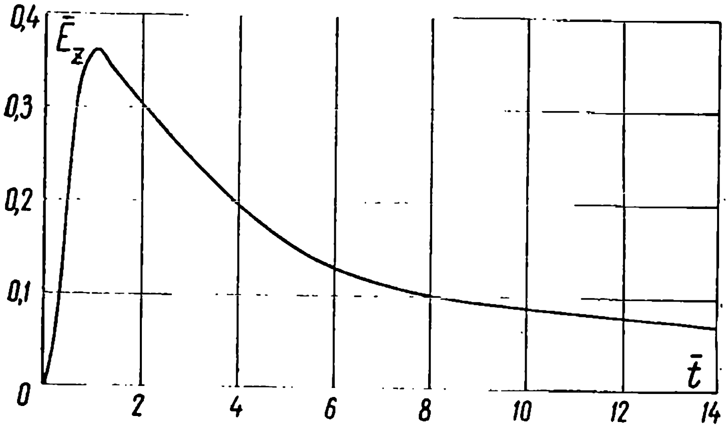

Figure 1

Curve Showing the Establishment of an Electric Field of an Infinitely Long Conductor in a Conductive Medium

where A_1, ϕ_1 are the potentials for the region $z>0$ and $\sigma \neq 0$, and A_{II}, ϕ_{II} are the potentials for $z<0$ and $\sigma = 0$. In addition to the boundary conditions (13), the vector potential must satisfy the condition of being limited at infinity, and must have a singularity of order $\frac{\mu_0 \Im}{4\pi r}$ at the origin (Ref. 31).

We shall try to find the solution of the problem which has been posited by the method of separation of variables. We shall assume the following /140

$$A_y = \frac{\mu_0 \Im\, dy}{4\pi} \int_0^\infty J_0(\lambda r)\, Y(z, \lambda, t)\, d\lambda \tag{14}$$

and

$$A_z = \frac{\mu_0 \Im\, dy}{4\pi} \cdot \frac{\partial}{\partial y} \int_0^\infty J_0(\lambda r)\, Z(z, \lambda, t)\, d\lambda, \tag{15}$$

where $r = \sqrt{x^2+y^2}$. It may be readily seen that the functions Y and Z satisfy the equation

$$\frac{\partial^2 U}{\partial z^2} - \lambda^2 U = \sigma\mu_0 \frac{\partial U}{\partial t}. \tag{16}$$

In view of the well known relationship (Ref. 31)

$$\int_0^\infty J_0(\lambda r)\, e^{-\lambda |z|} = \frac{1}{\sqrt{r^2 - z^2}}$$

it is clear that, in order that the vector potential have the requisite singularity at the origin, the following condition must be fulfilled

$$Y_{\mathrm{I}}|_{z=0}=Y_{\mathrm{II}}|_{z=0},\quad \left.\frac{\partial Y_{\mathrm{I}}}{\partial z}\right|_{z=0}-\left.\frac{\partial Y_{\mathrm{II}}}{\partial z}\right|_{z=0}=-2\lambda. \tag{17}$$

In view of the fact that Y_{II} satisfies the following equation in the case of $z<0$

$$\frac{\partial^2 Y_{\mathrm{II}}}{\partial z^2}-\lambda^2 Y_{\mathrm{II}}=0,$$

whose solution is given by the following function

$$Y_{\mathrm{II}}(z,\lambda,t)=u(\lambda,t)\,e^{\lambda z},$$

we find that

$$\left.\frac{\partial Y_{\mathrm{II}}}{\partial z}\right|_{z=0}=\lambda Y_{\mathrm{I}}|_{z=0}.$$

Consequently, in order that the first and second conditions of (17) be satisfied, it is necessary that

$$Y_{\mathrm{I}}|_{z=0}=u(\lambda,t), \tag{18}$$

$$\left.\frac{\partial Y_{\mathrm{I}}}{\partial z}\right|_{z=0}-\lambda Y_{\mathrm{I}}|_{z=0}=-2\lambda. \tag{19}$$

Thus, this problem may be reduced to the one-dimensional case of the third boundary value problem for an equation of the parabolic type. As is known (Ref. 32), the solution of the latter problem has the following form /141

$$Y(z,\lambda,t)=\frac{2\lambda}{\sqrt{\pi\sigma\mu_0}}\int_0^t\left\{e^{-\frac{\sigma\mu_0 z^2}{4(t-\tau)}}-\lambda\int_0^\infty e^{-\lambda\varsigma-\frac{\sigma\mu_0(z+\varsigma)^2}{4(t-\tau)}}d\varsigma\right\}\times \tag{20}$$
$$\times e^{-\frac{\lambda^2(t-\tau)}{\sigma\mu_0}}\frac{d\tau}{\sqrt{t-\tau}}.$$

With allowance for (14), the following expression is obtained for A_y

$$A_y=\frac{\mu_0\Im\,dy}{2\pi}\int_0^\infty J_0(\lambda r)\left[\frac{\lambda}{\sqrt{\pi\sigma\mu_0}}\int_0^t\left\{e^{-\frac{\sigma\mu_0 z^2}{4(t-\tau)}}-\right.\right. \tag{21}$$
$$\left.\left.-\lambda\int_0^\infty e^{-\lambda\varsigma-\frac{\sigma\mu_0(z+\varsigma)^2}{4(t-\tau)}}d\varsigma\right\}e^{-\frac{\lambda^2}{\sigma\mu_0}(t-\tau)}\frac{d\tau}{\sqrt{t-\tau}}\right]d\lambda.$$

The boundary conditions for the component A_z are obtained from div A_{II} = 0 and (13) or

$$\left.\frac{\partial A_{zII}}{\partial z}\right|_{z=0} = -\left.\frac{\partial A_{yII}}{\partial y}\right|_{z=0} \tag{22}$$

and

$$A_{zI}|_{z=0} = A_{zII}|_{z=0}.$$

With allowance for (15), expressions (22) lead to the following equation

$$\frac{\partial^2 Z}{\partial z^2} - \lambda^2 Z = \sigma\mu_0 \frac{\partial Z}{\partial t} \tag{23}$$

and the following boundary conditions

$$Z_1|_{z=0} = -\frac{1}{\lambda} u(\lambda, t) = -\frac{1}{\lambda} Y_1\Big|_{z=0}. \tag{24}$$

In order to find A_z, following the procedure given by A. N. Tikhonov (Ref. 4), let us determine the magnitude of the scalar potential as follows: /142

$$\begin{gathered}\varphi = \frac{1}{\sigma\mu_0} \operatorname{div} \mathbf{A} = \frac{\Im\, dy}{4\pi\sigma} \cdot \frac{\partial}{\partial y} \int_0^\infty J_0(\lambda r) \left[Y + \frac{\partial Z}{\partial z} \right] d\lambda = \\ = \frac{\Im\, dy}{4\pi\sigma} \cdot \frac{\partial}{\partial y} \int_0^\infty J_0(\lambda r)\, S\,(z, \lambda, t)\, d\lambda,\end{gathered} \tag{25}$$

where

$$S(z, \lambda, t) = Y(z, \lambda, t) + \frac{\partial Z}{\partial z}(z, \lambda, t). \tag{26}$$

It is apparent that S satisfies equation (16), the initial zero condition, and the following boundary condition

$$S|_{z=0} = -\frac{1}{\lambda} \cdot \left.\frac{\partial Y_1}{\partial z}\right|_{z=0} + Y_1\Big|_{z=0} = 2. \tag{27}$$

Employing the well known solution of the first boundary value problem for an equation of the parabolic type (Ref. 31), we obtain

$$S = \frac{1}{\sqrt{\pi\sigma\mu_0}} \int_0^t \frac{z}{(t-\tau)^{3/2}} \exp\left[-\frac{\sigma\mu_0 z^2}{4(t-\tau)} - \frac{\lambda^2}{\sigma\mu_0}(t-\tau) \right] d\tau. \tag{28}$$

Taking into account the following relationship (Ref. 35)

$$\int_0^\infty J_0(\lambda r)\, e^{-\frac{\lambda^2}{\sigma\mu_0}(t-\tau)}\, d\lambda = \frac{1}{2} \sqrt{\frac{\pi\sigma\mu_0}{t-\tau}}\, e^{-\frac{\sigma\mu_0 r^2}{8(t-\tau)}} I_0\left(\frac{\sigma\mu_0 r^2}{8(t-\tau)}\right),$$

we find that

$$\varphi=\frac{\Im\, dy}{8\pi\sigma}\cdot\frac{\partial}{\partial y}\int_0^t \frac{z}{(t-\tau)^2} e^{-\frac{\sigma\mu_0\left(z^2+\frac{1}{2}r^2\right)}{4(t-\tau)}} I_0\left(\frac{\sigma\mu_0 r^2}{8(t-\tau)}\right). \tag{29}$$

Omitting the very cumbersome calculations (Ref. 4), we shall only present certain results which will be of interest to us in the future.

The electric dipole field on the surface of the half-space (z = 0) is determined by the following expression:

$$\mathbf{E}|_{z=0}=\frac{\Im\, dy}{2\pi\sigma}\left\{\operatorname{grad}\frac{\partial}{\partial y}\left(\frac{1}{r}\right)-\frac{\mathbf{y}}{r^3}F(\bar{t})\right\}, \tag{30}$$

where /143

$$F(\bar{t})=\Phi\left(\frac{1}{\sqrt{\bar{t}}}\right)-\frac{2}{\sqrt{\pi}}\cdot\frac{1}{\sqrt{\bar{t}}}e^{-\frac{1}{\bar{t}}},\quad \bar{t}=\frac{4t}{\sigma\mu_0 r^2};$$

where y is the unit vector directed along the Y axis.

If the source of the field is not a dipole, but any line L which connects the points A and B, then the electric field of the line L is determined by the integral

$$\mathbf{E}=\int_L \mathbf{E}_s(x, y, 0, t;\ \xi, \eta)\,ds=$$
$$=\frac{\Im}{2\pi\sigma}\left\{\operatorname{grad}_{(x,y)}\left(\frac{1}{r_A}-\frac{1}{r_B}\right)-\int_L F(\bar{t})\,ds\right\}, \tag{31}$$

where

$$\mathbf{E}_s(x, y, 0, t;\ \xi, \eta)=-\frac{\Im\, ds}{2\pi\sigma}\left\{\operatorname{grad}_{(x,y)}\frac{\partial}{\partial s}\left(\frac{1}{r}\right)-\frac{\mathbf{s}}{r^3}F(\bar{t})\right\}$$

is the electric field of the dipole located at a point with the coordinates ξ, η(z = 0) in the s-direction.

When the conductor is located on the Y axis and is infinitely long, a simple formula may be obtained from (31) for a single component of the electric field

$$E_y=-\frac{\Im}{\pi\sigma x^2}\left(1-e^{-\frac{\sigma\mu_0 x^2}{4t}}\right), \tag{32}$$

and the scalar potential is ϕ = 0. We shall assume that the electric dipole directed along the Y axis is located at the origin on the surface of a conductive layer having the thickness ℓ. Assuming that A_x = 0, we find that the continuity conditions must be satisfied in the case of z = ℓ, in addition to the boundary conditions (13), i.e.,

$$A_{y\mathrm{II}}|_{z=l}=A_{y\mathrm{III}}|_{z=l},\quad A_{z\mathrm{II}}|_{z=l}=A_{z\mathrm{III}}|_{z=l}, \tag{33}$$

$$\frac{\partial A_{yII}}{\partial z}\Big|_{z=l} = \frac{\partial A_{yIII}}{\partial z}\Big|_{z=l}, \quad \varphi_{II}|_{z=l} = \varphi_{III}|_{z=l}, \tag{33}$$

where A_{II}, ϕ_{II} are the potentials for $0 \leqslant z \leqslant \ell$ and A_{III}, ϕ_{III} are the potentials for $z \geqslant \ell$.

Thus, as follows from (19), the influence of the region z<0 may be replaced by the boundary condition

$$\frac{\partial Y}{\partial z}\Big|_{z=0} - \lambda Y\Big|_{z=0} = -2\lambda. \tag{34}$$

Taking (33) into account, we readily find that the influence of the region $z \geqslant \ell$ may be replaced by the condition /144

$$\frac{\partial Y}{\partial z}\Big|_{z=l} + \lambda Y\Big|_{z=l} = 0. \tag{35}$$

In a similar way, we have

$$Z\Big|_{z=0} = -\frac{1}{\lambda} Y\Big|_{z=0} \tag{36}$$

and

$$Z\Big|_{z=l} = -\frac{1}{\lambda} Y\Big|_{z=l}. \tag{37}$$

It is clear from conditions (34) - (37) that the solution for a stratified medium may be reduced to the solution of the third boundary value problem of a one-dimensional equation of the parabolic type (16) with the boundary conditions (34) - (37). The solution of this problem was provided by A. N. Tikhonov and O. A. Skugarevskaya (Ref. 8-10), who employed two methods which yielded the value of the electric field over a wide range of time changes. We shall not go into a detailed examination of the calculations for solving this problem. We shall only try to illustrate by means of individual examples the general pattern of the methods employed by A. N. Tikhonov and O.A. Skugarevskaya, which provided a comprehensive representation of field propagation in stratified media.

In order to obtain the solution for the initial stage during which the field is established, it is advantageous to employ the reflection method (Ref. 8). As was shown by A. N. Tikhonov (Ref. 8) (for variable $\tau = \frac{t}{\sigma\mu_0}$), in order to obtain the solution of the equation

$$\frac{\partial^2 Y}{\partial z^2} - \lambda^2 Y = \sigma\mu_0 \frac{\partial Y}{\partial t} \tag{38}$$

with and initial zero condition and the boundary conditions (34) - (37), the function Y may be represented in the form of a series for small values of τ

$$Y(z, \lambda, \tau) = \sum_{n=0}^{\infty} [Y_{2n}(2nl + z, \lambda, \tau) - Y_{2n+1}(2(n+1)l - z, \lambda, \tau)] = \tag{39}$$

$$= Y_0(z,\lambda,\tau) + \sum_{n=0}^{\infty}[Y_{2(n+1)}(2(n+1)l+z,\lambda,\tau) - Y_{2n+1}(2(n+1)l-$$

$$-z,\lambda,\tau)] = Y_0(z,\lambda,\tau) - Y_1(2l-z,\lambda,\tau) + Y_2(2l+z,\lambda,\tau) - \ldots, \tag{39}$$

All the functions included in the right part of equality (39) are determined so that they satisfy the boundary conditions (34) - (37) as follows: /145

$$-\frac{\partial Y_0}{\partial z} + \lambda Y_0 = 2\lambda \tag{40}$$

and

$$-\frac{\partial Y_{2(n+1)}}{\partial z} + \lambda Y_{2(n+1)} = \frac{\partial Y_{2n+1}}{\partial z} + \lambda Y_{2n+1} \text{ for } z=0 \tag{41}$$

and

$$\frac{\partial Y_{2n+1}}{\partial z} + \lambda Y_{2n+1} = \frac{\partial Y_{2n}}{\partial z} + \lambda Y_{2n} \text{ for } z=l. \tag{42}$$

All the functions Y_m may be completely determined with these conditions. The function Y_0 may be found from the solution for a semi-limited line. The remaining functions may be found by means of the operator

$$L_\lambda(\varphi) = \int_z^\infty e^{\lambda(z-\zeta)}\varphi(\zeta)\,d\zeta \tag{43}$$

so that

$$Y_n(z,\lambda,\tau) = 2\lambda(-1+2\lambda L_\lambda)^n(1-\lambda L_\lambda)v_0. \tag{44}$$

v_0 is determined by means of a fundamental solution of the thermal conductivity equation, i.e.,

$$v_0 = 2\int_0^\tau G(z,\lambda,t)\,dt =$$

$$= \frac{1}{\sqrt{\pi}}\int_0^\tau \sqrt{\frac{1}{\tau-t}}\exp\left[-\frac{z^2}{4(\tau-t)} - \lambda^2(\tau-t)\right]dt. \tag{45}$$

As was shown by A. N. Tikhonov (Ref. 8), the convergence of the series (39) is determined by the condition

$$|Y_{2n}(2nl+z,t) - Y_{2n+1}(2(n+1)l-z,t)| \leqslant 9^n c e^{-\frac{(2nl)^2}{4t}}. \tag{46}$$

As may be seen directly from (46), the series converges particularly rapidly for small t. For the initial stage, it is absolutely permissible to confine oneself to the first three terms of series (39), whose values expressed by means of the operator L_λ are as follows:

$$Y_0 = 2\lambda v_0 - 2\lambda^2 L(v_0);$$
$$Y_1 = -2\lambda v_0 + 6\lambda^2 L(v_0) - 4\lambda^3 L^2(v_0); \tag{47}$$
$$Y_2 = 2\lambda v_0 - 10\lambda^2 L(v_0) + 16\lambda^3 L^2(v_0) - 8\lambda^4 L^3(v_0).$$

Following the procedure of A. N. Tikhonov, in order to determine the function S (26) let us introduce the auxiliary function W (z, λ, τ), so that /146

$$Z(z,\lambda,\tau) = -\frac{1}{\lambda} Y(z,\lambda,\tau) + \frac{2}{\lambda} W(z,\lambda,\tau). \tag{48}$$

According to conditions (36) (37), we find that W (z, λ, τ) satisfies the same equation as Y, with initial zero conditions and with the following conditions

$$W(0,\lambda,\tau) = 0; \quad W(l,\lambda,\tau) = Y(l,\lambda,\tau). \tag{49}$$

We are interested in the magnitude of the electric field on the surface of a layer, and consequently the quantity we desire is

$$S|_{z=0} = \left[Y + \frac{\partial Z}{\partial z}\right]_{z=0} = \left[Y - \frac{1}{\lambda}\cdot\frac{\partial Y}{\partial z} + \frac{2}{\lambda}\cdot\frac{\partial W}{\partial z}\right]_{z=0} = \\ = 2\left[1 + \frac{1}{\lambda}\cdot\frac{\partial W}{\partial z}\right]_{z=0}. \tag{50}$$

According to condition (27), the first component represents the value of S for $\ell = \infty$ (in the case of half-space), and the second represents the correction for the finiteness of the conductive layer thickness. Thus, the computation of S may be reduced to calculating the function W. Representing it in the form of series

$$W(z,\lambda,\tau) = \sum_{n=0}^{\infty} [W_{2n}(z,\lambda,\tau) - W_{2n+1}(z,\lambda,\tau)], \tag{51}$$

according to conditions (48) we find that

$$W_n(z,\lambda,0) = 0, \quad W_n(0,\lambda,\tau) = 0,$$
$$W_{2n}(l,\lambda,\tau) = Y_{2n}[(2n+1)l,\lambda,\tau],$$
$$W_{2n+1}(l,\lambda,\tau) = Y_{2n+1}[(2n+1)l,\lambda,\tau].$$

Consequently, the functions W_{2n} and W_{2n+1} are solutions of equation (38) with the conditions (34) - (37). It is apparent that they equal

$$W_{2n}(z,\lambda,\tau) = \sum_{k=0}^{\infty} \{Y_{2n}[2(n+k+1)l - z,\lambda,\tau] - \\ - Y_{2n}[2(n+k+1) + z,\lambda,\tau]\} \tag{52}$$

and /147

$$W_{2n+1}(z,\lambda,\tau) = \sum_{k=0}^{\infty} \{Y_{2n+1}[n+k+1)l - z,\lambda,\tau] - \\ - Y_{2n+1}[2(n+k+1)l + z,\lambda,\tau]\}. \tag{53}$$

As the first approximation, we may take

$$W(z,\lambda,\tau)=Y_0(2l-z,\lambda,\tau)-Y_0(2l+z,\lambda,\tau)-\\ -Y_1(2l-z,\lambda,\tau)+Y_1(2l+z,\lambda,\tau) \tag{54}$$

and

$$\left.\frac{\partial W}{\partial z}\right|_{z=0}=-2\left[\frac{\partial Y_0}{\partial z}-\frac{\partial Y_1}{\partial z}\right]_{z=0}. \tag{55}$$

For the desired function S, we obtain

$$S|_{z=0}=2-\frac{4}{\lambda}\left[\frac{\partial Y_0}{\partial t}-\frac{\partial Y_1}{\partial z}\right]_{z=0}. \tag{56}$$

Thus, the approximate value of S in the case of z = 0 may be expressed by means of Y_0 and Y_1.

The study (Ref. 9) presents detailed computations of the functions Y_0, which are employed to express the dependence of the electric field in which we are interested. In view of the cumbersome nature of the final formulas, we shall not present the end results here, which have no special interest for our purposes. We shall next investigate the solution of the problem for the final stage during which the field is established, which was given in (Ref. 10).

In order to find the function Y (z, λ, t) of our preceding problem in a form which is suitable for the investigation in case of large t, it may be conveniently represented in the following form

$$Y(z,\lambda,t)=Y^{(0)}(z,\lambda)+\bar{Y}(z,\lambda,t), \tag{57}$$

where $Y^{(0)}$ is the stationary solution, and $\bar{Y}$ is the deviation from the stationary solution. In order to determine these functions, according to (34) - (38) we have the following condition

$$\left.\begin{aligned}&\frac{\partial^2 Y^{(0)}}{\partial z^2}-\lambda^2 Y^{(0)}=0; \quad 0\leqslant z\leqslant l;\\ &\frac{\partial Y^{(0)}}{\partial z}-\lambda Y^{(0)}=-2\lambda \quad \text{for} \quad z=0;\\ &\frac{\partial Y^{(0)}}{\partial z}+\lambda Y^{(0)}=0 \quad \text{for} \quad z=l\end{aligned}\right\} \tag{58}$$

and /148

$$\left.\begin{aligned}&\frac{\partial^2 \bar{Y}}{\partial z^2}-\lambda^2\bar{Y}=\sigma\mu_0\frac{\partial\bar{Y}}{\partial t}; \quad 0\leqslant z\leqslant l;\\ &\frac{\partial\bar{Y}}{\partial z}-\lambda\bar{Y}=0 \quad \text{for} \quad z=0;\\ &\frac{\partial\bar{Y}}{\partial z}+\lambda\bar{Y}=0 \quad \text{for} \quad z=l;\\ &\bar{Y}(z,\lambda,0)=-Y^{(0)}(z,\lambda).\end{aligned}\right\} \tag{59}$$

It is apparent that

$$Y^{(0)}(z, \lambda) = e^{-\lambda z}.$$

Assuming

$$\overline{Y}(z, \lambda, t) = Y_n(z, \lambda) T_n(\lambda, t),$$

we find that

$$\frac{d^2 Y_n}{dz^2} + k_n^2 Y_n = 0, \tag{60}$$

$$\frac{\partial Y_n}{\partial z} - \lambda Y_n = 0 \text{ for } z = 0; \tag{61}$$

$$\frac{\partial Y_n}{\partial z} + \lambda Y_n = 0 \text{ for } z = l$$

and

$$T_n = Ce^{-\mu_n \tau},$$

where

$$\mu_n = k_n^2 + \lambda^2 \text{ and } \tau = \frac{t}{\sigma \mu_0}.$$

The general solution of equation (60) is

$$Y_n = C_1 \cos k_n z + C_2 \sin k_n z.$$

We find the following from the boundary conditions (61)

$$C_2 = \frac{\lambda}{k_n} C_1 \text{ and } \operatorname{tg} k_n l = \frac{2 k_n l}{k_n^2 - \lambda^2}.$$

Separating the right and left part of the latter equality for n = 0 in powers of λ and retaining terms up to the third order, we obtain

$$k_0^2(\lambda) = \frac{2}{l}\lambda - \frac{1}{3}\lambda^2 + \frac{2}{45} l \lambda^3. \tag{62}$$

The function $\bar{Y}$ may be represented in the following form /149

$$\overline{Y}(z, \lambda, \tau) = \sum_{n=0}^{\infty} C_n Y_n(z, \lambda) e^{-(k_n^2 + \lambda^2)\tau}. \tag{63}$$

The coefficients C_n are determined as Fourier coefficients of the initial function $Y^{(0)}$:

$$C_n = -\frac{\int_0^l Y^{(0)}(z, \lambda) Y_n(z, \lambda)\, dz}{\int_0^l [Y_n(z, \lambda)]^2 dz}. \tag{64}$$

Taking into account the values of $Y^{(0)}$ and Y_n, we readily find that

$$\int_0^l Y^{(0)}(z,\lambda)\,Y_n(z,\lambda)\,dz=\frac{2\lambda}{k_n^2+\lambda^2}. \tag{65}$$

With allowance for the expansion of (62) we obtain

$$\int_0^l [Y_0(z,\lambda)]^2dz\approx l+\frac{1}{3}l^2\lambda+\frac{2}{45}l^3\lambda^2. \tag{66}$$

Consequently,

$$C_0=-\frac{2\lambda}{(k_0^2+\lambda^2)\left(l+\frac{1}{3}l^2\lambda+\frac{2}{45}l^3\lambda^2\right)}, \tag{67}$$

and we obtain the asymptotic formula

$$Y(z,\lambda,\tau)\approx-\frac{2\lambda\left[\cos k_0z+\frac{\lambda}{k_0}\sin k_0z\right]}{(k_0^2+\lambda^2)\left(l+\frac{1}{3}l^2\lambda+\frac{2}{45}l^3\lambda^2\right)}e^{-(k_0^2+\lambda^2)\tau}, \tag{68}$$

which may be applied to the case of small values of λ, which we shall be interested in later on. In order to illustrate the use of the formula obtained, let us calculate the derivative $\frac{\partial A_y}{\partial t}$ in the case of $z = 0$: /150

$$\left.\frac{\partial A_y}{\partial t}\right|_{z=0}=\frac{\mu_0\Im dy}{4\pi}\int_0^\infty J_0(\lambda r)\,\dot{Y}(0,\lambda,t)\,d\lambda\approx$$

$$\approx\frac{\mu_0\Im dy}{4\pi}\int_0^\infty J_0(\lambda r)\,\frac{2\lambda\exp\left[-\left(\frac{2}{l}\lambda+\frac{2}{3}\lambda^2+\frac{2}{45}l\lambda^3\right)\tau\right]}{\left(l+\frac{1}{3}l^2\lambda+\frac{2}{45}l^3\lambda^2\right)}\,d\lambda. \tag{69}$$

The presence of the exponential factor $\exp\left(-\frac{2}{\ell}\lambda\tau\right)$ leads to the fact that the value of the integral for large t is determined by the value of the integrand for small λ, which substantiates the use of asymptotic formula (68). Decomposing the factor $J_0(\lambda r)\exp\left(-\frac{2}{\ell}\lambda\tau\right)$ under the integral sign in series with respect to λ and confining ourselves to three terms of the expansion we obtain

$$\frac{\partial A_y}{\partial t}\approx\frac{\mu_0\Im dy}{4\pi}\int_0^\infty J_0(\lambda r)\left[\frac{2}{l}\lambda-\frac{2}{3}\lambda^2+\frac{2}{3}\left(\frac{1}{5}l-\frac{2}{l}\tau\right)\lambda^3\right]e^{-\frac{2}{l}\lambda\tau}\,d\lambda= \tag{70}$$

$$=\frac{\mu_0 \Im dy}{4\pi}\left\{\frac{2}{l}\cdot\frac{2\tau}{l\left(r^2+\frac{4\tau^2}{l^2}\right)^{3/2}}-\frac{2}{3}\left[-\frac{\lambda}{\left(r^2+\frac{4\tau^2}{l^2}\right)^{3/2}}+\frac{12\tau^2}{l^2\left(r^2+\frac{4\tau^2}{l^2}\right)^{5/2}}\right]+\right.$$

$$\left.+\frac{2}{3}\left(\frac{l}{5}-\frac{2}{l}\tau\right)\left[-\frac{18\tau}{l\left(r^2+\frac{4\tau^2}{l^2}\right)^{5/2}}+\frac{120\tau^3}{l^3\left(r^2+\frac{4\tau^2}{l^2}\right)^{7/2}}\right]\right\}, \tag{70}$$

since

$$\int_0^\infty J_0(\lambda r)e^{-\alpha\lambda}d\lambda=\frac{1}{\sqrt{r^2+\alpha^2}}$$

and

$$\int_0^\infty J_0(\lambda r)\lambda^n e^{-\alpha\lambda}d\lambda=(-1)^n\frac{\partial^n}{\partial\alpha^n}\cdot\frac{1}{\sqrt{r^2+\alpha^2}}.$$

The function W may be determined as the solution of the boundary value /151
problem

$$\frac{\partial^2 W}{\partial z^2}-\lambda^2 W=\sigma\mu_0\frac{\partial W}{\partial t},$$

$$W|_{z=0}=0,\quad W|_{t=0}=0, \tag{71}$$

$$W|_{z=l}=Y|_{z=l}=e^{-\lambda l}+\sum_{n=0}^{\infty}C_n Y_n(l,\lambda)e^{-\mu_n\tau},$$

which leads to the asymptotic formula (Ref. 10)

$$W(z,\lambda,\tau)\approx e^{-\lambda l}\frac{\operatorname{sh}\lambda z}{\operatorname{sh}\lambda l}+C_0(\lambda)Y_0(\lambda l)\frac{\sin k_0 z}{\sin k_0 l}e^{-\mu_0\tau}. \tag{72}$$

We shall not continue the calculation for the electric field components on the surface of a conductive layer, and we shall thus conclude the examination of the methods developed by A. N. Tikhonov and O. A. Skugarevskaya. These methods were employed to obtain the relationship for the initial and final stage during which the field was established in a conductive layer lying on an ideally conducting base (Ref. 11-12), as well as to determine the process during which the field is established in a three-layered medium (Ref. 15). Figure 2 presents curves showing the dependence of $\bar{E}_y=\frac{\pi\sigma r^3}{\Im dy}E_y$ on $\bar{t}=\frac{4t}{\sigma\mu_0 r^2}$ taken from (Ref. 7), which were computed according to formulas for the electric field of a dipole lying on the surface of a conductive layer. Calculations according to formulas for the initial stage were performed for $\bar{t}\leqslant 0.5$, and for the final stage -- for $\bar{t}\geqslant 0.1$. The dashed curve presents the results derived from calculations for half-space. As may be seen from the curves, the presence of a limited layer leads to an increase in the field amplitude. This may be explained by reflection

of the field from the layer surface.

We may obtain the electric field of an infinitely long conductor, located on the Y axis, which is given by the simplest formulas, in the case of a conductive medium and half-space by integration of the expressions obtained for the dipole directed along the Y axis. In view of the complexity of the dipole expressions, it is not always advantageous to solve the problem for the dipole, and then to integrate over an infinitely long conductor. Since we have only one component of the vector potential A_y in the last case and since all quantities depend on the two coordinates x and y, we may regard the problem as a two-dimensional problem. In view of this fact, the scalar potential and the component A_z of the vector potential vanish. The component A_y which may be deter- /152 mined by expression (14), may be determined by the following transformation, as may be readily seen:

$$A_y = \frac{\mu_0 \mathfrak{J}}{4\pi} \int_0^\infty Y(z, \lambda, t) \cos \lambda x \frac{d\lambda}{\lambda}. \tag{73}$$

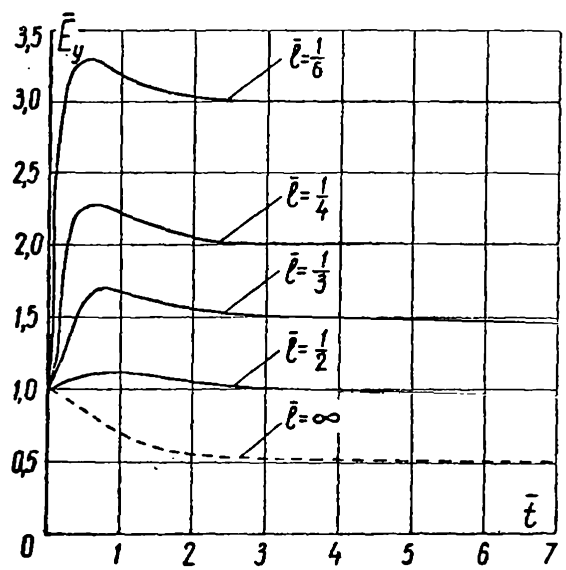

Figure 2

Curves Showing the Establishment of the Electric Dipole Field on a Conductive Layer Surface for Different $\bar{\ell} = \frac{\ell}{r}$, where ℓ is the Layer Thickness; r - is the Distance from the Dipole Along the X Axis.

The problem of determining A_y may be reduced to applying the methods which we investigated above for determining the function Y.

The methods developed by A. N. Tikhonov and O. A. Skugarevskaya, which are based on an analysis of the solution of the thermal conductivity equation and which are rather complex in nature, make it possible to analyze the solution graphically over a wide range of changes in time. In order to examine similar problems, other authors have employed an operational method (Ref. 1-2, 5, 6, 13, 14, 18-21, 27-29). As we shall subsequently see, the operational method may be visualized more clearly, as compared with the methods of A. N. Tikhonov. Difficulty is entailed here when we determine the inverse transform and analyze the solution. D. N. Chetayev (Ref. 13) has provided a graphic application of the Laplace transformation for solving the problem regarding the /153 establishment of the field in stratified media, when the conductivity of the two layers differs only slightly.

Let us now investigate the propagation of electromagnetic pulsed fields in moving media, where the operational

method will be illustrated at the same time.

3. Propagation of Pulsed Electromagnetic Fields in Moving Electroconductive Media

Let us begin the examination of the propagation of pulsed electromagnetic fields in moving media with the problem in which the source of the field is an electric dipole located at the origin. The vector potential of this dipole satisfies the following equation (Ref. 29)

$$\nabla^2\mathbf{A}-\sigma\mu_0(\mathbf{v}\nabla)\mathbf{A}-\sigma\mu_0\frac{\partial\mathbf{A}}{\partial t}=-\mu_0\mathbf{j}_0 \qquad (74)$$

and the initial condition

$$\mathbf{A}|_{t=0}=0, \qquad (75)$$

where $j_0 = \Im d\mathbf{r}\delta(\mathbf{r})$ and $\delta(r) = \delta(x)\delta(y)\delta(z)$ is the three-dimensional delta-function.

In order to simplify subsequent computations, we shall assume that the medium moves in the direction of the X axis. In this case, it is natural to assume that $v_x = v$, $v_y = v_z = 0$, $A_x = A_z = 0$ and A_y satisfy the following equation

$$\nabla^2 A_y-\sigma\mu_0 v\frac{\partial A_y}{\partial x}-\sigma\mu_0\frac{\partial A_y}{\partial t}=-\mu_0\Im dy\delta(\mathbf{r}). \qquad (76)$$

Applying the Laplace transformation

$$\overline{A}_y=\int\limits_0^\infty A_y e^{-pt}dt=LA_y \qquad (77)$$

to equation (76), with allowance for the initial conditions (75), we obtain

$$\Delta^2\overline{A}_y-\sigma\mu_0 v\frac{\partial\overline{A}_y}{\partial x}-\sigma\mu_0 p\overline{A}_y=-\frac{1}{p}\mu_0\Im dy\delta(\mathbf{r}). \qquad (78)$$

The solution of equation (78) may be written by means of the Green's function for a point source (Ref. 34):

$$\overline{A}_y=\frac{\mu_0\Im dy}{(2\pi)^3 p}\int\limits_{-\infty}^{\infty}dk_x\int\limits_{-\infty}^{\infty}dk_y\int\limits_{-\infty}^{\infty}\frac{e^{ikr}}{k^2+ik_x\sigma\mu_0 v+\sigma\mu_0 p}dk_z, \qquad (79)$$

where $kr = k_x x+k_y y+k_z z$ and $k^2 = k_x^2+k_y^2+k_z^2$. The inverse transform of the integrand may be conveniently written, by applying the multiplication theorem for the Laplace transformation (Ref. 30): /154

$$A_y=\frac{\mu_0\Im dy}{\sigma\mu_0\pi^3}\int\limits_0^\infty dk_x\int\limits_0^\infty dk_y\int\limits_0^\infty dk_z\int\limits_0^t e^{-\frac{k^2\tau}{\sigma\mu_0}}\cos k_x(x-v\tau)\times$$
$$\times\cos k_y y\cos k_z z d\tau. \qquad (80)$$

Integrating over k_x, k_y and k_z, we obtain the following expression

$$A_y=\frac{\Im dy}{\sigma}\int_0^t\left(\frac{\sigma\mu_0}{4\pi\tau}\right)^{\frac{3}{2}}\exp\left\{-\frac{\sigma\mu_0}{4\tau}[(x-v\tau)^2+y^2+z^2]\right\}d\tau, \tag{81}$$

which, as may be readily seen, may be reduced to the expression for the Hertz potential (7) in the case of v = 0, with allowance for (8).

The same result is obtained for the case when the dipole is directed along Z, and the motion takes place in the X direction, just as previously. For a dipole directed along X (i.e., in the direction of motion of the medium) we have

$$A_x=\frac{\Im dx}{\sigma}\int_0^t\left(\frac{\sigma\mu_0}{4\pi\tau}\right)^{\frac{3}{2}}\exp\left\{-\frac{\sigma\mu_0}{4\tau}[(x-v\tau)^2+y^2+z^2]\right\}d\tau. \tag{82}$$

The scalar potential may be determined by the following relationship

$$\varphi=\frac{1}{\sigma\mu_0}\operatorname{div}\mathbf{A}-v_x A_x. \tag{83}$$

In order to find the field of an arbitrary circuit or conductor, we must integrate the dipole potential over this circuit or conductor, assuming beforehand that it is formed of elementary dipoles, whose vector potential is determined by formulas (81) and (82).

By way of an example, let us calculate the electric field of a circular circuit in the plane (Y, Z) with the center at the origin. The vector potential of the dipole located in the plane (Y, Z), with the coordinates η, ς and the length ds, has the following form:

$$\mathbf{A}_s=\frac{\Im ds}{\sigma}\int_0^t\left(\frac{\sigma\mu_0}{4\pi\tau}\right)^{\frac{3}{2}}\exp\left\{-\frac{\sigma\mu_0}{4\tau}[(x-v\tau)^2+(y-\eta)^2+(z-\zeta)^2]\right\}d\tau. \tag{84}$$

Introducing the cylindrical coordinates /155

$$y=r\cos\psi,\quad z=r\sin\psi,$$
$$\eta=a\cos\alpha,\quad \zeta=a\sin\alpha,$$
$$ds=ad\alpha,$$

we obtain the expression for the single component A_ϕ of the vector potential of the circular circuit:

$$A_\varphi=\frac{\Im a}{\sigma}\int_0^t\left(\frac{\sigma\mu_0}{4\pi\tau}\right)^{\frac{3}{2}}\exp\left\{-\frac{\sigma\mu_0}{4\tau}[a^2+r^2+(x-v\tau)^2]\right\}\times$$
$$\times\int_{-\pi}^{\pi}\exp\left\{\frac{\sigma\mu_0}{2\tau}ar\cos(\alpha-\psi)\right\}\cos\alpha\,d\alpha\,d\tau. \tag{85}$$

In the case of cylindrical symmetry, we may set $\psi = 0$ and, employing the integral representation for the Bessel function of a purely imaginary arguement (Ref. 31), we obtain

$$A_\varphi = \frac{2\pi a \Im}{\sigma} \int_0^t \left(\frac{\sigma\mu_0}{4\pi\tau}\right)^{\frac{3}{2}} \exp\left\{-\frac{\sigma\mu_0}{4\tau}[a^2 + r^2 + (x - v\tau)^2]\right\} I_1\left(\frac{\sigma\mu_0 ar}{2\tau}\right) d\tau. \tag{86}$$

The scalar potential $\phi = 0$, and the following expression is obtained for the electric field

$$E_\varphi = -\frac{\partial A_\varphi}{\partial t} \doteq \frac{2\pi a \Im}{\sigma} \left(\frac{\sigma\mu_0}{4\pi t}\right)^{\frac{3}{2}} \exp\left\{-\frac{\sigma\mu_0}{4t}[a^2 + r^2 + (x - vt)^2]\right\} I_1\left(\frac{\sigma\mu_0 ar}{2t}\right). \tag{87}$$

The electromotive force, directed in the circular circuit of radius r which is located at the distance x from the first circuit, may be expressed by the integral

$$\mathfrak{E} = \int_0^{2\pi} E_\varphi r d\varphi = 2\pi r E_\varphi. \tag{88}$$

Consequently, we have

$$\overline{\mathfrak{E}} = -\left(\frac{1}{\bar{t}}\right)^{\frac{3}{2}} \exp\left\{-\frac{1}{\bar{t}}[1 + \bar{r}^2 + (\bar{x} - \bar{v}\,\bar{t})^2]\right\} I_1\left(\frac{2\bar{r}}{\bar{t}}\right), \tag{89}$$

where

$$\overline{\mathfrak{E}} = \frac{\sigma a^2}{r\sqrt{2\pi\Im}} \mathfrak{E}; \quad \bar{t} = \frac{4t}{\sigma\mu_0 a^2}; \quad \bar{r} = \frac{r}{a}; \quad \bar{x} = \frac{x}{a} \text{ and } \bar{v} = \frac{1}{4}\sigma\mu_0 a v$$

are dimensionless variables. The latter results in the case of r = a coincide with those obtained in (Ref. 27).

Let us present still another method for solving the problem for moving media when a solution of equation (2) is not required, and when we may confine ourselves to employing the Galilean transformation (Ref. 28, 29). This method is sometimes simpler, as compared with the solution of the problem with an equation for moving media (Ref. 27). In order to examine this method, let us introduce the four-dimensional potential Φ with the components (Ref. 33) /156

$$\Phi_1 = A_x, \quad \Phi_2 = A_y, \quad \Phi_3 = A_z, \quad \Phi_4 = \frac{i}{c}\varphi. \tag{90}$$

and the four-dimensional radius vector $\mathbf{x}$ with the components

$$x_1 = x, \quad x_2 = y, \quad x_3 = z, \quad x_4 = ict. \tag{91}$$

When changing from an inmobile system of reference to a system which moves in the x direction with the velocity v with respect to the first, according to the theory of relativity, the four-dimensional potential and the coordinates may satisfy the Lorentz transformation

$$\Phi'_1 = \frac{\Phi_1 + i\frac{v}{c}\Phi_4}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad \Phi'_2 = \Phi_2, \quad \Phi'_3 = \Phi_3, \tag{92}$$

$$\Phi'_4=\frac{\Phi_4-i\frac{v}{c}\Phi_1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{92}$$

and

$$x'_1=\frac{x_1+i\frac{v}{c}x_4}{\sqrt{1-\frac{v^2}{c^2}}},\quad x'_2=x_2,\quad x'_3=x_3,$$
$$x'_4=\frac{x_4-i\frac{v}{c}x_1}{\sqrt{1-\frac{v^2}{c^2}}}. \tag{93}$$

Examining the case of slowly moving media and disregarding terms containing ℓ/c^2, let us turn to the Galilean transformation

$$A'_x=A_x,\quad A'_y=A_y,\quad A'_z=A_z,$$
$$\varphi'=\varphi-vA_x \tag{94}$$

and /157

$$x'=x-vt;\quad y'=y;\quad z'=z;\quad t'=t. \tag{95}$$

Employing the transformation formulas obtained for electromagnetic potentials, we may readily obtain the solution for moving media. For this purpose, we must first solve the problem in a coordinate system moving together with the conductive medium, by transforming the potential and the coordinates of the immobile source of the field according to (94) (95). Employing the inverse transformation, we may then change to an immobile coordinate system.

Let us illustrate the statements presented above with the example of a two-dimensional problem (Ref. 28). Two infinitely long conductors, with different currents moving in the opposite direction and having the magnitude $\mathfrak{J}$, are arranged on the surface of moving half-space at the height h in the Y direction. The conductors are located symmetrically with respect to the YOZ plane. The XOY plane coincides with the surface of the half-space. The currents change according to a unit step function of time (4). In this case, the scalar potential vanishes, and only one component of the vector potential A_y = A remains, which satisfies the following equation in a system which is rigidly connected with the moving half-space:

$$\frac{\partial^2 A'}{\partial x'^2}+\frac{\partial^2 A'}{\partial z'^2}=\sigma\mu_0\frac{\partial A'}{\partial t'} \tag{96}$$

with the initial condition

$$A'(x',z',0)=0. \tag{97}$$

The continuity conditions at the boundaries z' = 0 and z' = h are the boundary conditions, just as previously. The derivative $\frac{\partial A'}{\partial z'}$ on the surface z' = h undergoes a discontinuity, which equals the density of the surface current j':

$$\begin{gathered} A'_{\mathrm{I}}(x', h, t') = A'_{\mathrm{II}}(x', h, t'), \\ \left.\frac{\partial A'_{\mathrm{II}}}{\partial z'}\right|_{z'=h} - \left.\frac{\partial A'_{\mathrm{I}}}{\partial z'}\right|_{z'=h} = \mu_0 j', \\ A'_{\mathrm{II}}(x', 0, t') = A'_{\mathrm{III}}(x', 0, t'), \\ \left.\frac{\partial A'_{\mathrm{II}}}{\partial z'}\right|_{z'=0} = \left.\frac{\partial A'_{\mathrm{III}}}{\partial z'}\right|_{z'=0}, \end{gathered} \tag{98}$$

where A'_{I} is the solution of equation (96) for the region $z' \geqslant h$, A'_{II} is the solution for $0 \leqslant z' \leqslant h$, and A'_{III} is the solution for $z' \leqslant 0$. In addition to the boundary conditions, we must take into account the limiting condition /158 for $z' \to \pm\infty$.

The density of the surface currents with respect to the immobile coordinate system may be expressed by means of the delta function

$$j = \mathfrak{J}[\delta(x-a) - \delta(x+a)] = \frac{2\mathfrak{J}}{\pi}\int_0^\infty \sin\lambda x \sin\lambda a d\lambda.$$

In the mobile coordinate system, we obtain the following by means of the transformation (95)

$$j' = \frac{2\mathfrak{J}}{\pi}\int_0^\infty \sin\lambda(x'+vt)\sin\lambda a d\lambda.$$

Applying the transformation (77) to equation (96) and boundary conditions (98), we obtain the equation

$$\frac{\partial^2\overline{A}'}{\partial x'^2} + \frac{\partial^2\overline{A}'}{\partial z'^2} = \sigma\mu_0 p\overline{A}' \tag{99}$$

and the boundary conditions

$$\begin{gathered} \overline{A}'_{\mathrm{I}}(x', h, p) = \overline{A}'_{\mathrm{II}}(x', h, p), \\ \left.\frac{\partial \overline{A}'_{\mathrm{II}}}{\partial z'}\right|_{z'=h} - \left.\frac{\partial \overline{A}'_{\mathrm{I}}}{\partial z'}\right|_{z=h} = \mu_0 \overline{j}', \\ \overline{A}'_{\mathrm{II}}(x', 0, p) = \overline{A}'_{\mathrm{III}}(x', 0, p), \\ \left.\frac{\partial \overline{A}'_{\mathrm{II}}}{\partial z'}\right|_{z'=h} = \left.\frac{\partial \overline{A}'_{\mathrm{III}}}{\partial z'}\right|_{z'=h}, \end{gathered} \tag{100}$$

where

$$\bar{j}' = \frac{2\mathfrak{I}}{\pi} \int_0^\infty F \sin \lambda a d\lambda$$

and

$$F = \frac{p \sin \lambda x' + \lambda v \cos \lambda x'}{p^2 + \lambda^2 v^2}.$$

Separating the variables of the last equation, we may readily obtain the solution satisfying the boundary conditions (100) and the limiting condition for $z' \to \pm\infty$.

After all the constants are determined, the solutions for all three regions assume the following form: /159

$$\bar{A}'_{\mathrm{I}} = \bar{A}'_{\mathrm{II}} = \bar{A}' = \frac{2\mu_0\mathfrak{I}}{\pi} \int_0^\infty F e^{-\lambda(z'+h)} \frac{d\lambda}{\lambda + \varkappa} - \tag{101}$$

$$- \frac{\mu_0\mathfrak{I}}{\pi} \int_0^\infty F e^{-\lambda(z'+h)} \frac{d\lambda}{\lambda} + \frac{\mu_0\mathfrak{I}}{\pi} \int_0^\infty F e^{-\lambda|z'-h|} \frac{d\lambda}{\lambda},$$

$$\bar{A}'_{\mathrm{III}} = \frac{2\mu_0\mathfrak{I}}{\pi} \int_0^\infty F e^{-\lambda h + \varkappa z'} \frac{d\lambda}{\lambda + \varkappa}, \tag{102}$$

where $\varkappa = \sqrt{\lambda^2 + \sigma\mu_0 p}$.

The inverse transform of the integrand may be found according to the well known formulas (Ref. 35) by employing the multiplication theorem:

$$A' = \frac{2\mu_0\mathfrak{I}}{\pi} \int_0^t d\tau \int_0^\infty e^{-\lambda(z'+h)} M(\lambda, \tau) \sin \lambda (x' + vt - v\tau) \sin \lambda a d\lambda - \tag{103}$$

$$- \frac{\mu_0\mathfrak{I}}{\pi} \int_0^\infty e^{-\lambda(z'+h)} \sin \lambda (x' + vt) \sin \lambda a \frac{d\lambda}{\lambda} +$$

$$+ \frac{\mu_0\mathfrak{I}}{\pi} \int_0^\infty e^{-\lambda|z'-h|} \sin \lambda (x' + vt) \sin \lambda a \frac{d\lambda}{\lambda},$$

$$A'_{\mathrm{III}} = \frac{2\mu_0\mathfrak{I}}{\pi} \int_0^\infty e^{-\lambda h} N(z', \lambda, \tau) \sin \lambda (x' + vt - v\tau) \sin \lambda a d\lambda, \tag{104}$$

where

$$M(\lambda, \tau) = \frac{e^{-\frac{\lambda^2\tau}{\sigma\mu_0}}}{\sqrt{\pi\sigma\mu_0\tau}} - \frac{\lambda}{\sigma\mu_0}\left[1 - \Phi\left(\sqrt{\frac{\lambda^2\tau}{\sigma\mu_0}}\right)\right]$$

and

$$N(z', \lambda, \tau) = \frac{\exp\left(-\frac{\sigma\mu_0 z'^2}{4\tau} - \frac{\lambda^2\tau}{\sigma\mu_0}\right)}{\sqrt{\pi\sigma\mu_0\tau}} -$$

$$-\frac{\lambda}{\sigma\mu_0} e^{\lambda z'} \left[1 - \Phi\left(\sqrt{\frac{\sigma\mu_0 z'^2}{4\tau}} + \sqrt{\frac{\lambda^2\tau}{\sigma\mu_0}}\right)\right].$$

Changing to the stationary coordinate system, according to (94), (95) we obtain /160

$$A = \frac{2\mu_0\mathfrak{I}}{\pi} \int_0^t d\tau \int_0^\infty e^{-\lambda(z+h)} M(\lambda, \tau) \sin\lambda(x - v\tau) \sin\lambda a d\lambda - \tag{105}$$

$$-\gamma(t) A_y^{\text{от}} + \gamma(t) A_y^{\text{ex}},$$

$$A_{\text{III}} = \frac{2\mu_0\mathfrak{I}}{\pi} \int_0^t d\tau \int_0^\infty e^{-\lambda h} N(z, \lambda, \tau) \sin\lambda(x - vt) \sin\lambda a d\lambda, \tag{106}$$

where

$$A_y^{\text{от}} = \frac{\mu_0\mathfrak{I}}{\pi} \int_0^\infty e^{-\lambda(z+h)} \sin\lambda x \sin\lambda a \frac{d\lambda}{\lambda} \tag{107}$$

is the expansion of the vector potential of the reflected currents and

$$A_y^{\text{ex}} = \begin{cases} \frac{\mu_0\mathfrak{I}}{\pi} \int_0^\infty e^{-\lambda(z-h)} \sin\lambda x \sin\lambda a \frac{d\lambda}{\lambda}, & z \geqslant h; \\ \frac{\mu_0\mathfrak{I}}{\pi} \int_0^\infty e^{\lambda(z-h)} \sin\lambda x \sin\lambda a \frac{d\lambda}{\lambda}, & z \leqslant h \end{cases} \tag{108}$$

is the expansion of the vector potential of the external currents.

The strength of the electric field for $z \geqslant 0$ is

$$E_y = -\frac{\partial A_y}{\partial t} - E_y^{\text{н}} + E_y^{\text{от}} = -E_y^{\text{ex}}, \tag{109}$$

where

$$E_y^{\text{н}} = \frac{2\mu_0\mathfrak{I}}{\pi} \int_0^\infty e^{-\lambda(z+h)} M(\lambda, t) \sin\lambda(x - vt) \sin\lambda a d\lambda, \tag{110}$$

$$E_y^{\text{от}} = \delta(t) A_y^{\text{от}}; \quad E_y^{\text{ex}} = \delta(t) A_y^{\text{ex}}$$

and $\delta(t)$ is the impulse function.

We shall omit the very cumbersome calculations of the integral (110), and we shall only present the final result for the field of the induced currents:

$$\bar{E}_y^{н} = \frac{w\left(\frac{\xi_1+i\zeta}{\sqrt{\bar{t}}}\right)-1}{(\xi_1+i\zeta)^2} + \frac{w\left(\frac{-\xi_1+i\zeta}{\sqrt{\bar{t}}}\right)-1}{(-\xi_1+i\zeta)^2} - \frac{w\left(\frac{\xi_2+i\zeta}{\sqrt{\bar{t}}}\right)-1}{(\xi_2+i\zeta)^2} - \tag{111}$$

$$- \frac{w\left(\frac{-\xi_2+i\zeta}{\sqrt{\bar{t}}}\right)-1}{(-\xi_2+i\zeta)^2} - \frac{2\zeta}{\sqrt{\pi\bar{t}}}\left(\frac{1}{\xi_1^2+\zeta^2} - \frac{1}{\xi_2^2+\zeta^2}\right),$$

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where

$$w(z) = e^{-z^2}\left(1 + \frac{2i}{\sqrt{\pi}}\int_0^z e^{u^2}du\right)$$

is the probability integral of the complex argument which is a tabulated function (Ref. 36);

$$\bar{x} = \frac{x}{a}; \quad \bar{z} = \frac{z}{a}; \quad \bar{t} = \frac{4t}{\sigma\mu_0 a^2}; \quad \bar{h} = \frac{h}{a};$$

$$\bar{v} = \frac{1}{4}\sigma\mu_0 a v; \quad \bar{E}_y^{н} = \frac{2\pi\sigma a^2}{\mathfrak{J}} E_y^{н}; \quad \zeta = \bar{z} + \bar{h}.$$

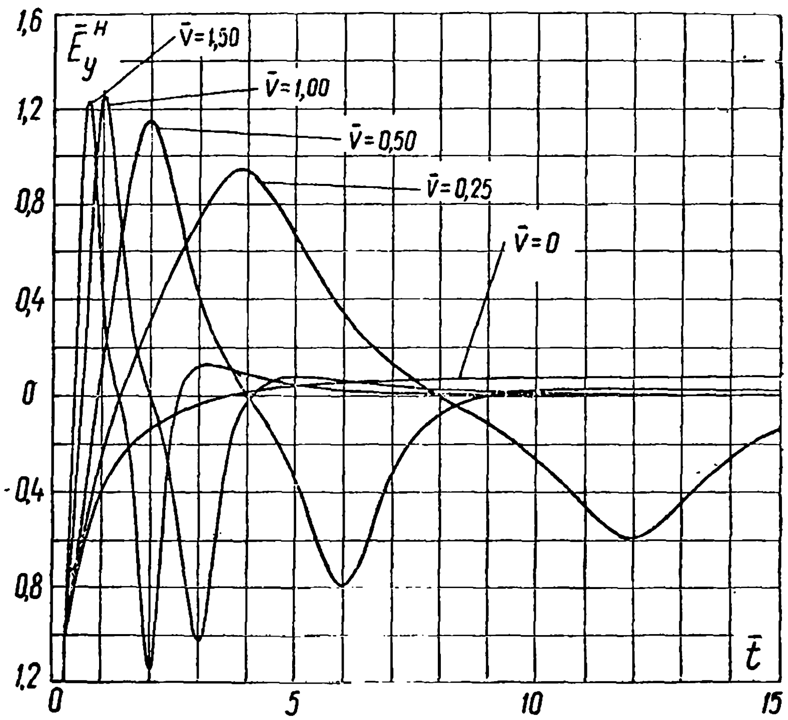

Figure 3

Dependence of the Function $\bar{E}_y^H$ on $\bar{t}$ and $\bar{v}$ in the Case of $\bar{x} = 2$ and $\varsigma = 0.5$

For the special when $\varsigma=0$ we have /162

$$\bar{E}_y^{\scriptscriptstyle H}=2\left[\frac{1}{\xi_1^2}\exp\left(-\frac{\xi_1^2}{\bar{t}}\right)-\frac{1}{\xi_2^2}\exp\left(-\frac{\xi_2^2}{\bar{t}}\right)\right]. \tag{112}$$

We may obtain the latter formula by means of (32), summing up the field of the two conductors and employing the substitution $x \to x - \upsilon t$. Figure 3 presents curves showing the dependence on $\bar{t}$ and $\bar{\upsilon}$, computed according to formula (111) for $\bar{x} = 2$ and $\varsigma = 0.5$.

As the last examples have shown, when allowance is made for the transformations (94) (95), we may readily change from computations which are performed for a conductive layer to computations for a moving conductive layer. For the case of an infinitely long conductor, these results are only achieved by replacing $x \to x - \upsilon t$.

In conclusion, we would like to point out that the transition of the function (111) through zero between two main maxima (see Figure 3) is determined by the condition $x = \upsilon t$. It may be readily seen that the electromotive force induced in a rectangular coil, which is arranged vertically with respect to the surface of half-space or the moving layer, satisfies this condition. This fact has been employed to compile a new method for measuring the velocity of conductive media (Ref. 25); this method is based on the transition through zero of the voltage in the measuring coil.

REFERENCES

1. Vvedenskiy, B.A. Toki Fuko pri aperiodicheskikh protsessakh v zheleze (Foucault Currents During Aperiodic Processes in Iron). Zhurnal Russkogo Fiziko-Khimicheskogo Obshchestva 55, 1, 1923.
2. Ollendorf, F. Erdstrome (Earth Currents). Berlin, 1928.
3. Ollendorf, F. Elektrische Schaltstrome in der Erde (Electrical Switching Currents in the Earth). Elektrische Nachrichten Technik, 5, 1928.
4. Tikhonov, A.N. O stanovlenii elektricheskogo toka v odnorodnom provodyashchem poluprostranstve (Establishment of an Electrical Current in a Uniform Conductive Half-Space). Izvestiya AN SSR, Seriya Geogr. i Geofiz 3, 1946.
5. Horton, C.W. On the Use of Electromagnetic Waves in Geophysical Projecting. Geophysics, 11, 1946.
6. Sheyman, S.M. Ob ustanovlenii elektromagnitnykh poley v zemle (Establishment of Electromagnetic Fields in the Earth). V Kn: Prikladnaya Geofizika (In the book: Applied Geophysics) 3, Moscow, 1947.
7. Tikhonov, A.N., Skugarevskaya, O.A. O stanovlenii elektricheskogo taka v neodnorodnoy sloistoy srede (Establishment of an Electric Current in a Non-Uniform, Stratified Medium). Izvestiya AN SSR, Seriya Geofiz. 6, 1951.
8. Tikhonov, A.N. O tret'yey krayevoy zadache dlya uravneniya parabolicheskogo tipa (The Third Boundary Value Problem for an Equation of the Parabolic Type). Izvestiya AN SSR, Seriya Geogr. i Geofiz. 3, 1950.
9. Tikhonov, A.N. O stanovlenii elektricheskogo toka v neodnorodnoy sloistoy srede (Establishment of an Electric Current in a Non-Uniform Stratified Medium). Ibid. 3, 1950.

10. Tikhonov, A.N., Skugarevskaya, O.A. O stanovlenii elektricheskogo toka v neodnorodnoy sloistoy srede (Establishment of an Electric Current in a Non-Uniform Stratified Medium), II. Ibid. 4, 1950.
11. Skugarevskaya, O.A. O nachal'noy stadii protsessa stanovleniya elektricheskogo toka v sloye, lezhashchem na ideal'no provodyashchem osnovanii (Initial Stage in the Process of Establishing an Electric Current in a Layer Lying on an Ideally Conducting Base). Izvestiya AN SSSR, Seriya Geofiz. 6, 1951.
12. Skugarevskaya, O.A. O konechnoy stadii protsessa stanovleniya elektricheskogo toka v sloye, lezhashchem na ideal'no provodyashchem osnovanii (Final Stage in the Process of Establishing an Electric Current in a Layer Lying on an Ideally Conducting Base). Izvestiya AN SSSR, Seriya Geofiz. 6, 1951. /163
13. Chetayev, D.N. K raschetu neustanovivshikhsya elektromagnitnykh poley v neodnorodnykh sredakh (Calculation of Non-Stationary Electromagnetic Fields in Non-Uniform Media). Trudy Geofiz. Instituta AN SSR. 32, 1956.
14. Datsev, A.B. Ob obshechey lineynoy zadache toploprovodnosti mnogosloynoy sredy (General Linear Problem of Thermal Conductivity of a Multi-Layered Medium). Izvestiyz AN SSR, Seriya Geogr. i Geofiz. 2, 1950.
15. Skugarevskaya, O.A. Raschet konechnoy stadii protsessa stanovleniya elektricheskogo polya v trekhsloynoy srede (Calculation of the Final Stage of the Process Establishing an Electric Field in a Triple Layered Medium). Izvestiya AN SSSR, Seriya Geofiz. 1, 1959.
16. Tikhonov, A.N., Skugarevskaya, O.A. Asimptoticheskoye povedeniye protsessa stanovleniya elektromagnitnogo polya (Asymptotic Behavior of the Process Establishing an Electromagnetic Field). Izvestiya AN SSSR, Seriya Geofiz. 6, 1959.
17. Tikhonov, A.N., Skugarevskaya, O.A. Ob asimptoticheskom povedenii protsessa stanovleniya elektromagnitnogo polya v sloistykh trassakh (Asymptotic Behavior of the Process Establishing an Electromagnetic Field in Stratified Traces). Izvestiya AN SSSR, Seriya Geofiz. 7, 1959.
18. Wait, J.R. Transient Electromagnetic Propagation in a Conducting Medium. Geophysics, 16, 1951.
19. Bhattacharyya, B.K. Propagation of an Electric Pulse through a Homogeneous and Isotropic Medium. Geophysics, 22, 1957.
20. Wait, J.R. Propagation of Electromagnetic Pulses in a Homogeneous Conducting Earth. Appl. Sci. Res. B, 8, p. 213, 1960.
21. Mijnarends, P.E. Propagation of Electromagnetic Step Function Over a Conducting Medium. J. Appl. Phys., 33, 8, 1962.
22. Novikov, V.V. Obzor rabot po rasprostraneniyu impul'snykh elektromagnitnykh signalov v provodyashchikh sredakh i nad zemnoy poverkhnost'yu (Review of Articles on Propatation of Pulsed Electromagnetic Signals in Conductive Media and Above the Earth's Surface). Problemy difraktsii i rasprostraneniya radiovoln (Problems of Diffraction and Radio Wave Propagation), II. Leningrad, 1962.
23. Zaborovskiy, A.I. Elektrorazvedka (Electrical Prospecting). GINTI, Moscow, 1963.
24. Sermons, G.Ya., Kalin', R.K., Ginzburg, A.S. Avtorskoye svidetel'stvo (Author's certificate), No. 168906, February 16, 1963.
25. Sermons, G.Ya., Ginzburg, A.S. Avtorskoye svidetel'stvo (Author's certificate), No. 166514, July 27, 1963.

26. Zheygur, B.D., Sermons, G.Ya. Impul'snyy metod izmereniya skorosti techeniya elektroprovodyashchey zhidkosti (Pulse Method of Measuring the Flow Rate of a Electroconductive Liquid). Magnitnaya Gidrodinamika 1, 1965.
27. Sermons, G.Ya. Rasprostraneniye impul'sa elektromagnitnogo polya v dvizhushcheysya elektroprovodyashchey srede (Propagation of an Electromagnetic Field Pulse in a Moving Electroconductive Medium). Izvestiya AN Latv. SSR, Seriya Fizicheskikh i Tekhicheskikh Nauk 1, 1964.
28. Semons, G.Ya. Rasprostraneniye impul'sa elektromagnitnogo polya vdol' dvizhushchegosya elektroprovodyashchego poluprostranstva (Propagation of an Electromagnetic Field Pulse Along a Moving, Electroconductive Half-Space). Izvestiya AN Latv. SSR, Seriya Fizicheskikh i Teckhicheskikh Nauk 4, 1964.
29. Sermons, G.Ya., Zheygur, B.D. Issledovaniya po rasprostraneniyu impul'sa elektromagnitnogo polya v dvizhushcheysya srede (Research on Propagation of an Electromagnetic Field Pulse in a Moving Medium). V kn: Voprosy Magnitnoy Gidrodinamiki (In the book: Problems of Magnetic Hydrodynamics), 4. Izdatel'stvo AN Latv. SSR, 1964.
30. Lavrent'yev, M.A., Shabat,B.A. Metody teorii funktsiy kompleksnogo peremennogo (Methods Involved in the Theory of the Function of a Complex Variable). GIFNL, 1958.
31. Tikhonov, A.N., Samarskiy, A.A. Uravneniya matematicheskoy fiziki (Equations of Mathematical Physics). Gosudarstvennoye Izdatel'stvo Tekhnicheskoy i Teoreticheskoy Literature, 1951.
32. Carslaw, H.S., Jaeger, D. Operational Methods in Applied Mathematics. Izdatel'stvo Inostrannoy Literatury, 1948.
33. Sommerfeld, A. Electrodynamics. Izdatel'stvo Inostrannoy Literature, 1958.
34. Ivanenko, D.I., Sokolov, A.A. Klassicheskaya teoriya polya (Classical Theory of a Field). Gosudarstvennoye Izdatel'stvo Tekhnicheskoy i Teoreticheskoy Literatury, 1951.
35. Ditkin, V.A., Kuznetsov, P.I. Spravochnik po operatsionnomu ischisleniyu (Handbook on Operational Calculus). Gosudarstvennoye Izdatel'stvo Tekhnicheskoy i Teoreticheskoy Literature, 1951.
36. Faddeyeva, V.N., Terent'yev, N.M. Tablitsy znacheniy integrala veroyatnostey ot kompleksnogo argumenta (Tables of Values for the Probability Integral of a Complex Argument). Moscow, 1954.

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